String Form Factors and Gauge Theory Correlators

Tristan Mc Loughlin

Trinity College Dublin

Based on work with T. Klose.

A (very brief) Overview

In the context of the AdS/CFT duality the usual superconformal symmetry allows one to find a number of interesting results valid at all values of the coupling.

However certain special theories possess an underlying integrable structure that allows the exact calculation of many more quantities including those which are not protected by supersymmetry.

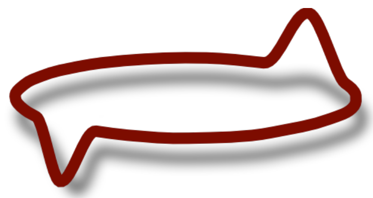
A (very brief) Overview

The canonical example:

$$\mathcal{N} = 4 \text{ SYM in 4d} \longleftrightarrow \text{Superstrings on } \mathrm{AdS}_5 \times S^5$$

in the planar or large N_c limit, where we consider single trace operators or correspondingly single closed strings described by a super-coset model,

$$\mathrm{Tr}(ZZZYZ\ldots ZYZZ) \longleftrightarrow$$



there are additional hidden symmetries [Bena, Polchinski & Roiban].
The spectrum of operator dimensions/string energies

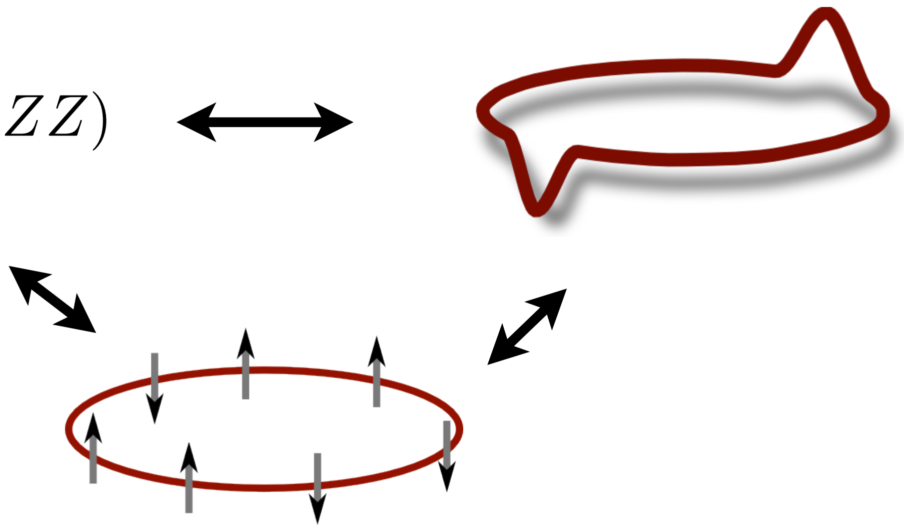
$$\mathfrak{D} \cdot \mathcal{O}^{(Q;n)} = \Delta(\lambda, Q; n)\mathcal{O}^{(Q;n)}$$

can be mapped to the eigenvalues of the Hamiltonian for an integrable system.

A (very brief) Overview

At weak coupling it is useful to map the perturbative calculation of operator dimensions to that of the energies of an integrable spin-chain [Minahan & Zarembo] [Beisert & Staudacher] :

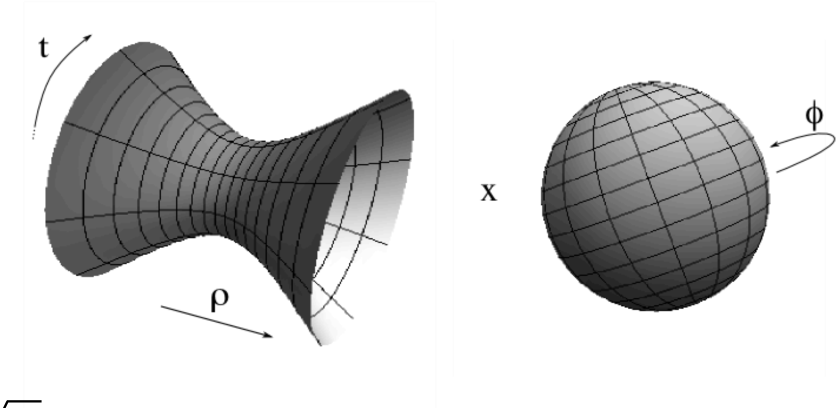
$$\mathrm{Tr}(ZZZYZ\ldots ZYZZ)$$



However at strong coupling, and indeed finite coupling, perhaps the clearest picture is given by the world-sheet light-cone gauge fixed string theory.

AdS$_5$ x S^5 string theory

Starting point is the GS string on AdS$_5$ x S^5 [Metsaev & Tseytlin]:



$$\mathcal{S} = -\frac{\sqrt{\lambda}}{4\pi}\int d^2\sigma\sqrt{-h}h^{\alpha\beta}G_{MN}\partial_a X^M\partial_b X^N + \text{fermions}$$

- Diffeomorphism and Weyl invariant
  - Local fermionic kappa-symmetry
    - Use (uniform) light-cone gauge

$$\tilde{\tau}(\tau,\sigma) = X^+ \equiv (1-a)t + a\phi\ , \quad p_- = (1-a)p_\phi - ap_t = \text{constant}$$

Light-cone world-sheet theory

- Theory is a massive, non-linear, non-Lorentz invariant $1+1$-dimensional theory defined on a cylinder:

$$\frac{L}{2\pi} = (1-a)\mathcal{J} + a\mathcal{E}$$

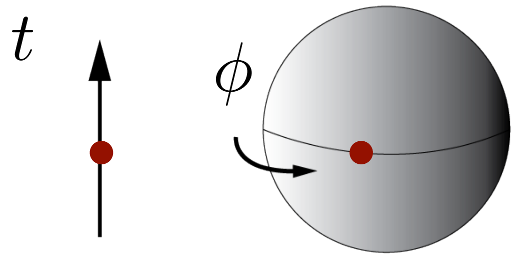
$$J = \sqrt{\lambda}\mathcal{J}$$

$$E = \sqrt{\lambda}\mathcal{E}$$

- There are 8 bosonic and 8 fermionic types of fundamental excitations

$$\Phi_i = \{Z_{\alpha\dot{\alpha}}, Y_{a\dot{a}}, \Psi_{a\alpha}, \Upsilon_{\alpha\dot{a}}\}$$

- Vacuum corresponds to point-like BMN string:

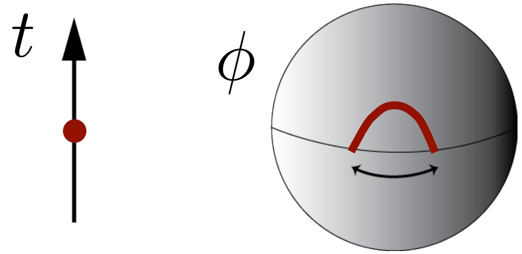


$$\mathbb{R} \times S^2 \subset \mathrm{AdS}_5 \times S^5$$

- Symmetries of the vacuum: $PSU(2|2)^2 \ltimes \mathbb{R}^3$ i.e. there is a central extension that corresponds to the world-sheet momentum - only non-zero in the decompactified limit.

De-compactified world-sheet theory

- In the de-compactified limit $L \to \infty$ one can consider excited states with non-vanishing total world-sheet momentum e.g. finite p magnons



with single particle energies:

$$E^2 = 1 + 4g^2 \sin^2 \frac{p}{2} \ , \qquad g = \frac{\sqrt{\lambda}}{2\pi}$$

- This world-sheet dispersion relation is uniformized in terms of Jacobi elliptic functions:

$$p = \mathrm{am}\ z \ , \qquad E = \mathrm{dn}(z, k) \ , \qquad k = -4g^2 < 0$$

and is so defined on the rapidity torus with periods

$$2\omega_1 = 4K(k) \ , \qquad 2\omega_2 = 4iK(1-k) - 4K(k)$$

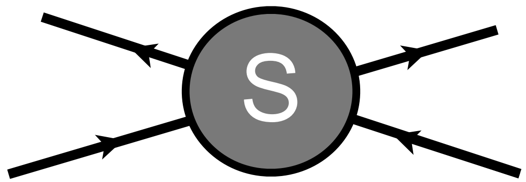
World-sheet Scattering

In the de-compactified limit one can define asymptotic incoming states and so "in"-scattering states

$$|p_1, p_2, \ldots, p_n\rangle^{(\text{in})}_{i_1, i_2, \ldots, i_n} \equiv |p_1, p_2, \ldots, p_n\rangle_{i_1, i_2, \ldots, i_n}$$

with $p_1 > p_2 > \cdots > p_n$ and reversed for "out"-scattering states.

The "in"- and "out"-scattering states are then related by the S-matrix

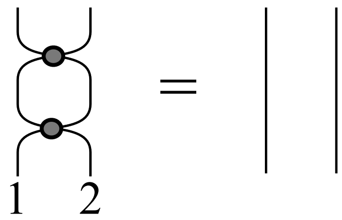


$$|p_1, p_2\rangle^{(\text{in})} = S_{12}(p_1, p_2) \cdot |p_1, p_2\rangle^{(\text{out})}$$

Integrable S-matrix

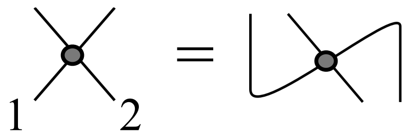
As an integrable theory the S-matrix satisfies several properties or axioms

Unitarity:



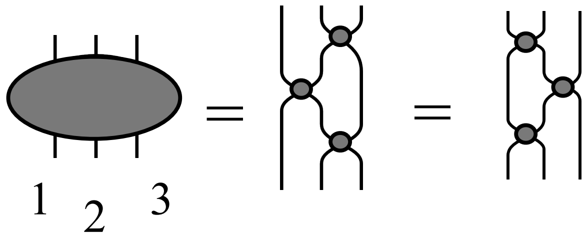
or $S_{12}(p_1,p_2).S_{21}(p_2,p_1)=\mathbb{I}$

Crossing:



or $S_{12}(p_1,p_2)=C_{(1)}S_{2\bar{1}}(p_2,-p_1)C^{-1}_{(1)}$

Yang-Baxter:



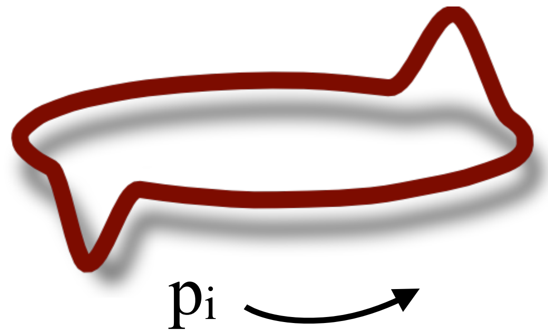
or $S_{12}S_{13}S_{23}=S_{23}S_{13}S_{12}$

Combined with a knowledge of the symmetries and bound states this allows for a complete determination of the S-matrix.

Anomalous Dimensions

For "long" single trace operators/infinite volume string states we can solve for the spectrum by means of the asymptotic Bethe ansatz.

A given operator is corresponds to a string asymptotic state with excitations of momenta $\{p_i\}$, we now reimpose periodic boundary conditions



$$e^{ip_i L} = \prod_{i \neq j} S(p_i, p_j) \quad \& \qquad E_{\rm Tot} = \sum_i E(p_i)$$

For short states we can also find all order results (though more technically complicated) [Gromov, Kazakov, and Vieira][Bombardelli, Fioravanti, and Tateo][Arutyunov and Frolov]

Some recent developments

More recently there have been notable related developments in (among other things):

- Improved methods for calculating the spectrum efficiently e.g. Quantum Spectral Curve.

Some recent developments

More recently there have been notable related developments in (among other things):

- Improved methods for calculating the spectrum efficiently e.g. Quantum Spectral Curve.
  - Initial calculations could be done numerically for specific operators using TBA equations [Gromov, Kazakov, Vieira] [Frolov] .
  - TBA is an infinite set of non-linear integral equations, can be reformulated in terms (via Y-system/Hirota T-system) as a finite set of non-linear Riemann-Hilbert equations [Gromov, Kazakov, Leurent, Volin] called QSC.
  - Very efficient for doing calculations which can be expanded to produce impressive perturbative results: e.g. Konishi to 9-loops [Marboe,Volin].
  - Provide checks on other conjectures e.g. BFKL predictions for twist-2 operators at 6-loops.

Some recent developments

More recently there have been notable related developments in (among other things):

- Improved methods for calculating the spectrum efficiently e.g. Quantum Spectral Curve.
- Using integrability to calculate other observables - scattering amplitudes, Wilson loops, Gauge theory form factors.
    - e.g. [Basso, Sever, Vieira] wrote a set of functional equations which allow them to compute null polygonal Wilson loops/scattering amplitude at finite coupling.

Some recent developments

More recently there have been notable related developments in (among other things):

- Improved methods for calculating the spectrum efficiently e.g. Quantum Spectral Curve.
- Using integrability to calculate other observables - scattering amplitudes, Wilson loops, Gauge theory form factors.
- Less supersymmetric dualities.
  - Strings on AdS_3 geometries with 16 susy have been shown to be classically integrable [Babichenko, Stefanski, Zarembo], [Sundin, Wulff]. This includes geometries with mixed RR and NSNS three-form flux [Cagnazzo, Zarembo]. The all-order BA have been written down [Borsato, Ohlsoson-Sax, Sfondrini Stefanski, Torrielli].
  - Progress on $AdS_2 \times S^2 \times T^6$: classically integrable [Sorokin, Tseytlin, Wulff, Zarembo][Cagnazzo,Sorokin, Wulff], exact S-matrix [Hoare, Pittelli, Torrielli].
  - q-deformed $AdS_5 \times S^5$ [Delduc, Magro,Vicedo], Jordanian Deformations [Kawaguchi, Matsumato, Yoshida] (includes gravity dual of NC theories of Maldacena-Russo, Hashimoto-Itzhaki)

Some recent developments

More recently there have been notable related developments in (among other things):

- Improved methods for calculating the spectrum efficiently e.g. Quantum Spectral Curve.
- Using integrability to calculate other observables - scattering amplitudes, Wilson loops, Gauge theory form factors.
- Less supersymmetric dualities.
- Three-point correlation functions.

CFT correlation functions

For a conformal field theory, symmetry is sufficient to fix the space-time dependence of the one-, two- and three-point correlation functions. E.g. scalar primary operators,

$$\langle \mathcal{O}_a(x_1)\mathcal{O}_b(x_2)\rangle = \frac{\delta_{a,b}}{|x_1 - x_2|^{2\Delta}}$$

$$\langle \mathcal{O}_a(x_1)\mathcal{O}_b(x_2)\mathcal{O}_c(x_3)\rangle = \frac{c_{abc}}{|x_{12}|^{\Delta_a+\Delta_b-\Delta_c}|x_{23}|^{\Delta_b+\Delta_c-\Delta_a}|x_{31}|^{\Delta_c+\Delta_a-\Delta_b}}$$

The non-trivial dependence on the coupling comes via the anomalous dimension and the structure constants. In principle all other correlation functions can be determined via the OPE.

$N = 4$ SYM three-point functions

It an obvious problem to try to extend our methods to the calculations of such structure constants. Currently there are several different approaches:

- Using spin-chain methods at low-loop orders. [Okuyama, Tseng] [Alday, David, Gava, Narain], more recently [Escobedo, Gromov, Sever, Vieira]. An important role is played by the scalar products of Bethe states.

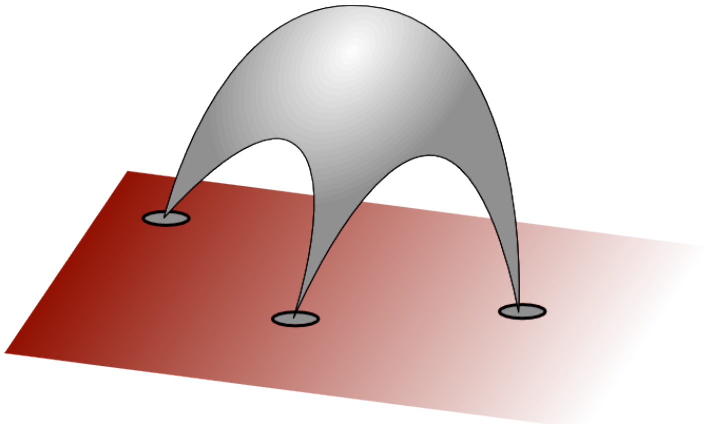
N = 4 SYM three-point functions

It an obvious problem to try to extend our methods to the calculations of such structure constants. Currently there are several different approaches:

- Using spin-chain methods at low-loop orders.
- We can think of three-point function in the dual string theory as:

$$\langle \mathcal{O}_a \mathcal{O}_b \mathcal{O}_c \rangle = \int \mathcal{D}\mathbb{X} \; \mathcal{V}_a(\mathbb{X}) \mathcal{V}_b(\mathbb{X}) \mathcal{V}_c(\mathbb{X}) \mathrm{e}^{-\sqrt{\lambda} S[\mathbb{X}]}$$

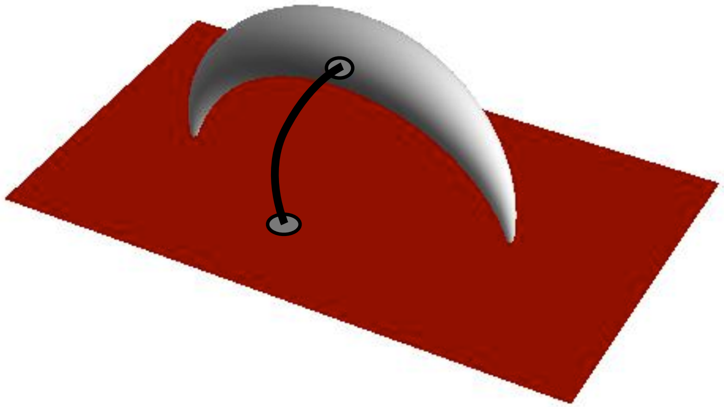
At strong coupling, for three "heavy" operators with large charges, we can try to evaluate by saddle point approximation:



- Don't know a lot of vertex operators.
- Difficult to find solutions however integrability can help [Janik, Surowka, Wereszczynski] [Kazama, Komatsu]

N = 4 SYM three-point functions

There is a class of three-point functions for which progress is easier: two-operators with large charges, one light with fixed charges e.g. BPS state corresponding to a massless (supergravity) scalar. [Zarembo] [Costa, Monteiro, Santos, Zoakos].



Given the classical solution, $\mathbb{X}_{cl}(\tau, \sigma)$, corresponding to the two heavy operators and the vertex operator, $\mathcal{V}_{\mathrm{L}}$, for the light operator the gauge theory structure constants at strong coupling are:

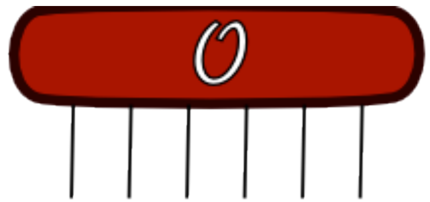
$$c_{\mathrm{HHL}} = const \cdot \int d\tau d\sigma \; \mathcal{V}_L[\mathbb{X}(\tau, \sigma)]$$

Important modifications due to [Bajnok,Janik, Wereszczynski].

World-sheet Form Factors

We are interested in calculating matrix elements of world-sheet operators between asymptotic states.

We define auxiliary functions as matrix elements for "in"-scattering state (i.e. with $p_1 > p_2 > \ldots > p_n$)



$$f_{\underline{i}}^{\mathcal{O}}(p_{\underline{i}}) = \langle \Omega | \mathcal{O} | p_1, \ldots, p_n \rangle_{i_1, \ldots, i_n}^{(\text{in})}$$

and for other values of the external momentum by analytic continuation

Can find other matrix elements by using crossing.

World-sheet Form Factors

- Form factors are basic observables of the world-sheet theory. As for any integrable model they important in understanding the theory beyond just the spectrum of energies.

- We are most interested in the case when the world-sheet operators are related to string vertex operators which create some state dual to a "short" gauge theory operator while the asymptotic states will correspond to generically non-BPS operators.

- In this case we conjecture the world-sheet form factors are related to gauge theory structure constants.

- We can see this by considering the spin-chain description of tree-level structure constants.

Spin-chain form factors

$$c^0_{abc} \propto \langle \uparrow\uparrow \downarrow \uparrow \ldots \uparrow |\sigma_+| \uparrow \downarrow \uparrow \downarrow \ldots \rangle$$

At weak coupling we can use spin-chain form factors to calculate structure constants at tree-level for the case where two operators have the same length.

Spin-chain form factors

Ex. [Roiban & Volovich] Operators made from single traces of two types of complex scalars

$$\mathcal{O} \sim \sum \cos \frac{\pi n(2s+1)}{L_c} \mathrm{Tr}\left[Y(Z)^s Y(Z)^{L_c-2-s}\right]$$

at one-loop this is described by XXX-spin chain

$$H = \frac{\lambda}{16\pi^2} \sum_{x=1}^{L_c} (1 - \vec{\sigma}_x \cdot \vec{\sigma}_{x+1})$$

Energy eigenstates:

$$|\psi(p_1, p_2, \ldots)\rangle = \frac{1}{\mathcal{N}} \sum \chi(p_1, p_2, \ldots)_{x_1 x_2 \ldots} |x_1, x_2, \ldots\rangle$$

where

$$|x_1, x_2, \ldots\rangle = S_{-,x_1} S_{-,x_2} \ldots |\uparrow \ldots \uparrow\rangle$$

and e.g. $\chi(p_1, p_2)_{x_1,x_2} = e^{-\frac{i}{2}\theta_{12}} e^{i(p_1 x_1 + p_2 x_2)} + e^{\frac{i}{2}\theta_{12}} e^{i(p_2 x_1 + p_1 x_2)}$

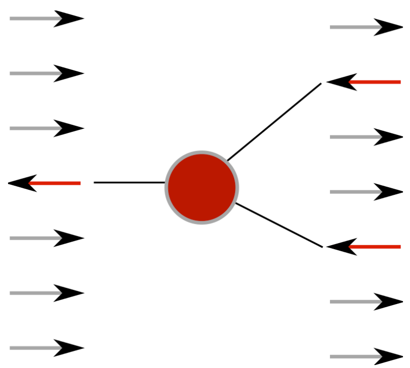
Spin-chain form factors

For the gauge theory/spin-chain operator

$$\mathcal{O}_b = \mathrm{Tr}(Z\bar{Y}) \leftrightarrow \mathcal{O}_b = \sum_{j=1}^{L_c} S_{+,j}$$

Consider

$$\mathcal{O}_a \sim \mathrm{Tr}(YZ^{L_c-1})$$



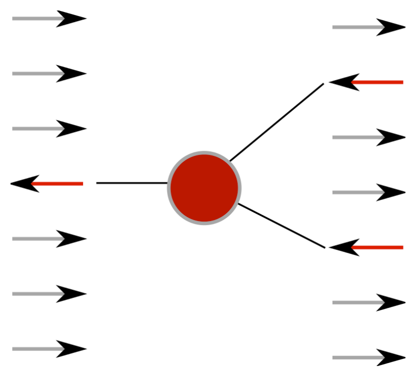
$$O^c \sim \sum_j \cos\frac{\pi n(2j+1)}{L_c-1}\mathrm{Tr}(YZ^jYZ^{L_c-2-j})$$

$$c^0_{abc} \sim f_{\mathrm{spin}}(\bar{p}_1, p_2, p_3) = {}_{L_c}\langle p_1|S_{+,j}|p_2,p_3\rangle_{L_c}$$

Let momenta satisfy B.E. but relax level matching
a.k.a trace cyclicity

Weak-strong matching

Example: in the limit $L_c \to \infty$ so that $p_i = \frac{2\pi n_i}{L_c} + \mathcal{O}(1/L_c^2)$ then for generic momenta and to leading order



$$\langle \psi(p_1) | S_{+,1} | \psi(p_2, p_3) \rangle_{\text{conn}} = \frac{2n_1(n_2 + n_3 - n_1)}{(n_1 - n_2)(n_2 - n_3)}$$

Can this be matched with string theory?

Weak-strong matching

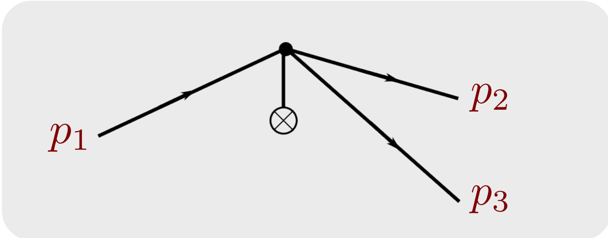
Procedure:

- Match spin-chain operator to string theory operator.
- Calculate form factor in the de-compactified limit.
- Re-compactify form factor by
  - imposing Bethe-Yang equations.
  - including correct normalisation (c.f. [Pozsgay and Takacs] finite-volume form factors).

For $\sqrt{\lambda} \gg 1$ the string action to quartic order:

$$\mathcal{L} = \partial_\alpha Y \partial^\alpha \bar{Y} - Y\bar{Y} + 2Y\bar{Y}\partial_\sigma Y \partial_\sigma \bar{Y} + \frac{1-2a}{2}((\partial Y)^2(\partial\bar{Y})^2 - Y^2\bar{Y}^2) + \ldots$$

For the operator $\mathcal{O} = Y$



$$f^Y(\bar{p}_1, p_2, p_3) = -2\frac{(p_2+p_3)^2 - (1-2a)(\mathbf{p_1}\cdot\mathbf{p_{123}}\mathbf{p_2}\cdot\mathbf{p_3}+1)}{\sqrt{8\epsilon_1\epsilon_2\epsilon_3}(\mathbf{p_{123}^2}-1)}$$

- Choose $a = 1$ gauge which gives the right identification of string and spin chain lengths to this order.
- Non-linear field redefinition (found from map to LL-model):

$$S_+ \sim \sqrt{2} Y e^{i\varphi}\left[1 - \frac{3}{2}|Y|^2 + \ldots\right]$$

- Normalise states (constant factor $1/L^{3/2}$)
- Final result matches.

Do we expect this agreement to persist? No.

This is in the double scaling limit:

$$f_{\rm string}(\lambda, J)\Big|_{\substack{\lambda\to\infty \\ J\to\infty \\ \tilde{\lambda}\to 0}} \stackrel{?}{=} f_{\rm spin}(\lambda, J)\Big|_{\substack{\lambda\to 0 \\ J\to\infty \\ \tilde{\lambda}\to 0}}$$

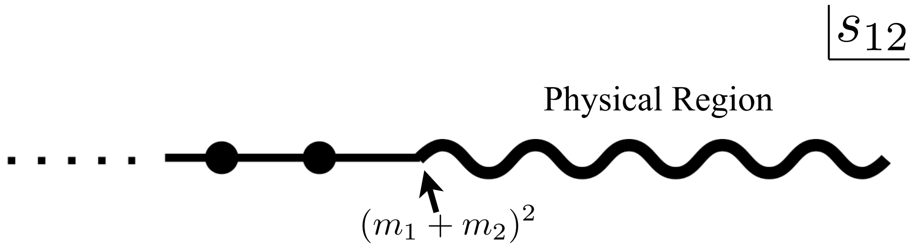
So agreement is due to some unknown non-renormalization theorem (presumably the same as that which gives matching of anomalous dimensions).

To get proper matching one should find all order answers.

Integrable Form Factors

This may be possible as due to unitarity, analyticity and locality there are a system of functional equations which when combined with integrability might allow form factors for this theory to be exactly calculable

Ex. Two-particle generalised form-factor in a Lorentz invariant, integrable theory



$$F^{\mathcal{O}}_{i_1,i_2}(s_{12} + i\epsilon) = \langle \Omega | \mathcal{O} | p_1, p_2 \rangle^{(\mathrm{in})}_{i_1,i_2} = \sum_{\mathrm{out}} \langle \Omega | \mathcal{O} | \mathrm{out} \rangle \langle \mathrm{out} | p_1, p_2 \rangle^{(\mathrm{in})}_{i_1,i_2}$$

$$= F^{\mathcal{O}}_{j_1,j_2}(s_{12} - i\epsilon) S^{j_1 j_2}_{i_1 i_2}(p_1, p_2) \ , \quad s_{12} \geq 4m^2$$

- Watson's equation

Such properties can be generalised and formalized as a set of axioms in terms of Zamolodchikov-Faddeev-algebra [Smirnov].

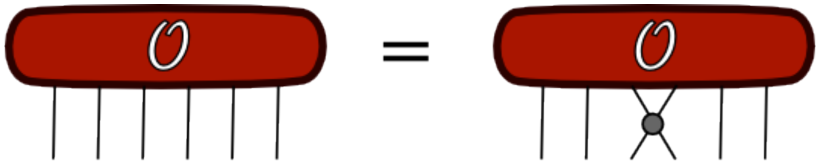
Form factor Axioms

Define

$$f^{\mathcal{O}}(z_1,\ldots,z_n) = \langle\Omega|\mathcal{O}|p(z_1),\ldots p(z_n)\rangle \ , \quad p(z_1) > \cdots > p(z_n)$$

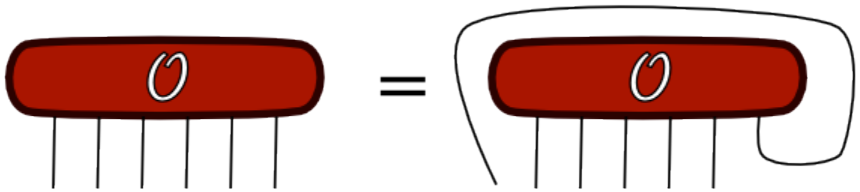
<u>Axioms</u>

i) Permutation:



$$f^{\mathcal{O}}(\ldots,z_j,z_i,\ldots) = f^{\mathcal{O}}(\ldots,z_i,z_j,\ldots)S(z_i,z_j)$$

ii) Periodicity:



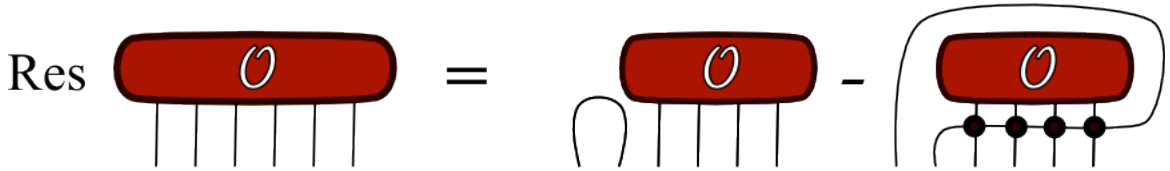
$$f^{\mathcal{O}}(z_1,z_2,\ldots,z_n) = f^{\mathcal{O}}(z_2,\ldots,z_n,z_1-2\omega_2)$$

Form factor Axioms

Define

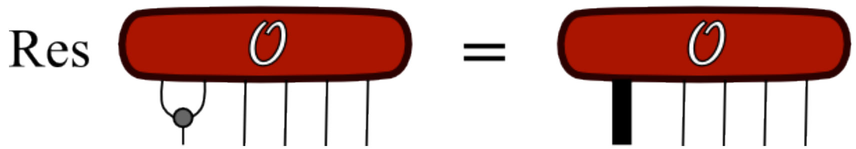
$$f^{\mathcal{O}}(z_1,\ldots,z_n) = \langle\Omega|\mathcal{O}|p(z_1),\ldots p(z_n)\rangle\ ,\quad p(z_1) > \cdots > p(z_n)$$

iii) One-particle poles: for $\mathbf{p}_1(z_1) + \mathbf{p}_2(z_2) = 0$



$$\text{Res}\ \ f^{\mathcal{O}}(z_1,\ldots,z_n) = 2iC_{12}f^{\mathcal{O}}(z_3,\ldots,z_n)(1 - S_{2n}\ldots S_{23})$$

iv) Bound state poles: for $\text{Res}_{z_1',z_2'}S_{12}(z_1,z_2) = R_{(12)}$



$$\text{Res}\ f^{\mathcal{O}}(z_1,z_2,\ldots,z_n) = \sqrt{2iR_{(12)}}f^{\mathcal{O}}(z_{12},z_3\ldots,z_n)$$

These "axioms" are obviously similar to relativistic case but are they correct? - Check perturbatively: i)-iii) are satisfied by the near-BMN perturbative answers.

Further checks:

At strong coupling, $\sqrt{\lambda} \gg 1$, we study the theory after performing a large boost with parameter, $\lambda^{1/4}$, such that the right-movers decouple and become free (Maldacena-Swanson limit):

$$\mathcal{L} = \partial_\alpha Y \partial^\alpha \bar{Y} - Y\bar{Y} + \gamma Y\bar{Y}(\partial_- Y)(\partial_- \bar{Y}) + \ldots$$

here focusing on a single complex boson.

Checked i)-iii) to two-loops.

Can't check iv) - existence of bound states - perturbatively but is apparent at "strong" coupling in spin-chain description.

Exact relativistic form factors

Ex. Consider two-particle form factors in relativistic theory. Answer can be written as

$$f^{\mathcal{O}}(\theta) = \mathcal{N}^{\mathcal{O}} K^{\mathcal{O}}(\theta) f_{\min}(\theta) \quad \text{s.t.} \qquad f_{\min}(\theta + 2\pi i) = S(-\theta) f_{\min}(\theta)$$

K is a even, periodic function which reproduces the pole structure and *N* is normalisation constant.

Remains to find a **minimal** solution i.e. one with no poles or zeros in the physical strip.

Exact relativistic form factors

Soln:

$$f_{\min}(\theta) = \prod_{n=1}^{\infty} S(-\theta + 2\pi i n)$$

We can make sense of this expression if we have an appropriate integral expression for the S-matrix:

First we Fourier transform the log of the S-matrix

$$h(t) = \frac{1}{\pi^2} \int_{-\infty}^{\infty} d\theta \;\; \log S(\theta) e^{\frac{i\theta t}{\pi}}$$

then it is straightforward to show

$$S(\theta) = e^{\int_0^\infty h(t) \sinh\left(\frac{\theta t}{i\pi}\right) dt} \Rightarrow f_{\min} = e^{-\int_0^\infty h(t) \frac{\cosh t\left(\frac{\theta}{i\pi} - 1\right)}{2 \sinh t} dt}$$

E.g. sine-Gordon Breather-Breather [Karowski and Weisz]

$$h(t) = \frac{2\cosh(\nu - \frac{1}{2})t}{t \cosh t/2}$$

World-sheet minimal solution

For the world-sheet theory, defined on a rapidity **torus** with periods $2\omega_1$ & $2\omega_2$, the minimal solution in a rank-one sector should satisfy:

$$f_{\min}(z_1 + 2\omega_2, z_2) = S(z_1, z_2) f_{\min}(z_1, z_2)$$

We will additionally assume that the form factors satisfy

$$f_{\min}(z_1 + \omega_2, z_2 + \omega_2) = f_{\min}(z_1, z_2)$$

which is essentially world-sheet parity (or analogue of CT transformation in relativistic model). Hence using $z_\pm = z_1 \pm z_2$ the equations we want to solve are

$$f_{\min}(z_+ + 2\omega_2, z_-) = f_{\min}(z_+, z_-)$$

$$f_{\min}(z_+, z_- + 2\omega_2) = S(z_+, z_-) f_{\min}(z_+, z_-)$$

An exact minimal solution

As in the relativistic case we can write the formal solution

$$\log f_{\min}(z_+, z_-) = \sum_{n=1}^{\infty} \log S(z_+, z_- - 2\omega_2 n)$$

and as before we can perform the sum by Fourier transforming S-matrix along real axis

$$\log S(z_-, z_+) = \sum_{m=-\infty}^{\infty} h(z_+, m) e^{\frac{i\pi m z_-}{\omega_1}}$$

$$\Rightarrow \log f_{\min}(z_-, z_+) = \sum_{n=1}^{\infty} h(z_+, n) \cos\left(\frac{n\pi(z_- - \omega_2)}{\omega_1}\right) \operatorname{csch}\left(\frac{i\pi n\omega_2}{\omega_1}\right)$$

Not particularly useful answer, to be more concrete we can expand and compare perturbative answers.

Conclusions

- Form factors appear to satisfy "axioms" very similar to those in relativistic integrable models from which exact, all-order answers can be found.

- Can be related to spin-chain form factors and so to tree-level gauge theory structure constants.

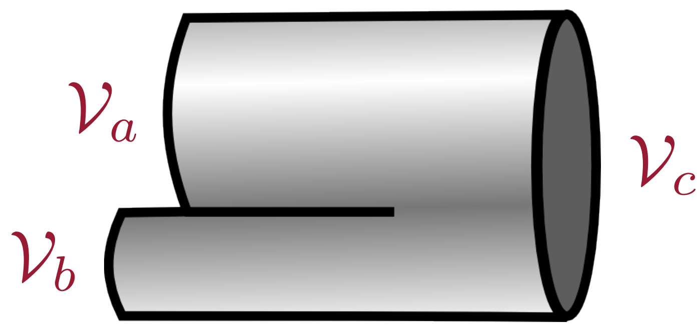
Outlook

Can we use the form factors to compute all order structure constants?

- For HHL states conjecture of [Bajnok, Janik and Wereszczyński]: structure constants correspond to diagonal finite volume form factors of the vertex operator of the light operator e.g. 2 two-magnon operators:

$$c_{HH\mathcal{O}} = \frac{f^{\mathcal{O}}(p_1,p_2) + L f^{\mathcal{O}}(p_2) + L f^{\mathcal{O}}(p_1)}{L + \phi_{12} + \phi_{21}}$$

Form factors as a step toward LCSFT for AdS/CFT space and so all three-point functions:



Thank You!