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Two-point functions in AdS/dCFT and the conformal bootstrap equations

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Introduction

- Recent progress has enabled the perturbative calculations of correlation functions of a non-trivial defect CFT related to $\mathcal{N}=4$ sYM.
- This dCFT has a holographic dual: The D3-probe-D5 brane setup with flux. The two-point functions then allow for tests of the AdS/dCFT correspondence as well as the application of integrability techniques in a less symmetric setup.

Correlators, Data and OPEs of dCFTs

- A *defect conformal field theory* (dCFT) is a scale-invariant theory with a spatial defect (boundary, interface, etc.). Consider a codimension one surface at $x_3=0$.
- The correlation functions are still highly constrained by the residual conformal symmetry, such that

$$\langle \mathcal{O}_i(x)\rangle = \frac{a_i}{x_3^{\Delta_{\mathcal{O}}}}\,, \quad \langle \mathcal{O}_i(x)\mathcal{O}_j(y)\rangle = \frac{f_{ij}(\xi)}{x_3^{\Delta_i} y_3^{\Delta_j}}$$
$$\langle \mathcal{O}_i(x)\hat{\mathcal{O}}_j(y)\rangle = \frac{\mu_{ij}}{x_3^{\Delta_i-\Delta_j}(x_3^2+|\mathbf{x}-\mathbf{y}|^2)^{\Delta_j}} \tag{1}$$

where $\xi = \frac{|x-y|^2}{4x_3y_3}$ is a conformally invariant cross ratio.

- One chooses the normalisation far from the defect

$$\lim_{\xi\to 0}\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\rangle = \frac{\delta_{ij}}{|x-y|^{2\Delta_i}} \tag{2}$$

- In addition to the ordinary *operator product expansion* (OPE)

$$\mathcal{O}_i(x)\mathcal{O}_j(y) = \sum_k \frac{\lambda_{ijk}}{|x-y|^{\Delta_{ijk}}} C(x-y,\partial_y)\mathcal{O}_k(y) \tag{3}$$

there is a *boundary operator expansion* (BOE)

$$\mathcal{O}_i(x) = \sum_k \frac{\mu_{ik}}{x_3^{\Delta_i-\Delta_k}} D(x_3,\partial_{\mathbf{x}})\hat{\mathcal{O}}_k(\mathbf{x}) \tag{4}$$

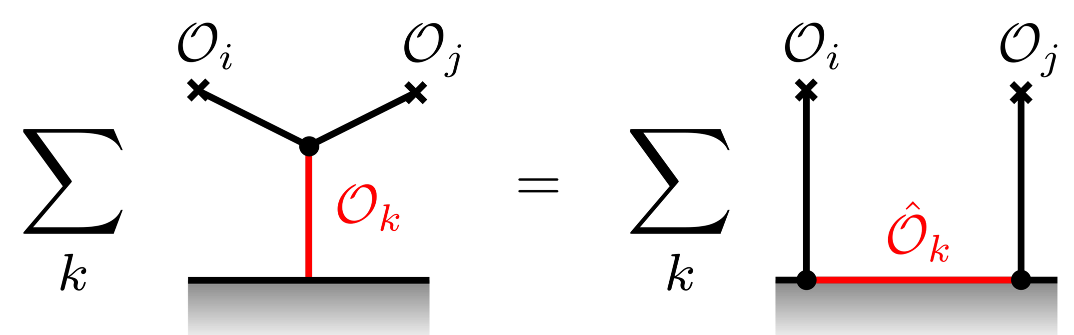
Boundary Conformal Bootstrap Equations

- The two-point functions has two conformal block decompositions

$$f_{ij}(\xi) = \xi^{-\frac{\Delta_i+\Delta_j}{2}}\sum_k \lambda_{ijk} a_k F_{\text{bulk}}(\Delta_k,\xi) = \sum_k \mu_{ik}\mu_{jk} F_{\text{bdy}}(\Delta_k,\xi) \tag{5}$$

where the bulk/boundary conformal blocks $F_{\text{bulk / bdy}}$ are known functions.

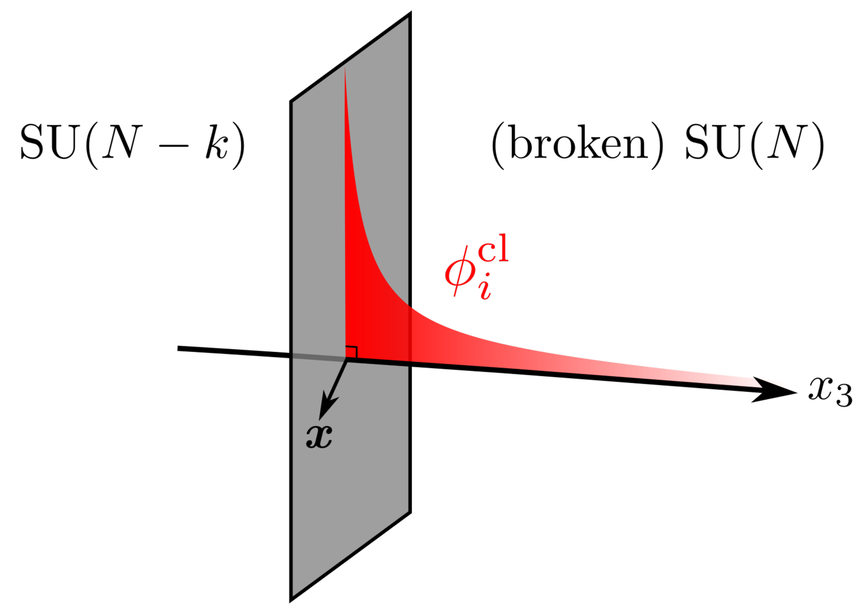
- This gives the crossing or *boundary conformal bootstrap* equation



A dCFT from $\mathcal{N}=4$ sYM

- Introduce a 1/2 BPS co-dimension one defect at $x_3=0$ interfacing between regimes with gauge group $SU(N-k)$ and $SU(N)$ for $\mathcal{N}=4$ sYM.
- For $x_3>0$ the gauge theory is Higgsed with scalars having a vev

$$\phi_i^{\text{cl}} = -\frac{1}{x_3} t_i^{(k)} \oplus 0_{(N-k)\times(N-k)} \tag{6}$$



where $[t_i,t_j] = i\epsilon_{ijk}t_k$.

- Expanding around the classical field gives a perturbative quantum description with x_3-dependent masses. The propagators can be written in terms of an AdS propagator

$$K(x,y) = \frac{g_{\text{YM}}^2}{2}\frac{K_{\text{AdS}}(x,y)}{x_3y_3} \tag{7}$$

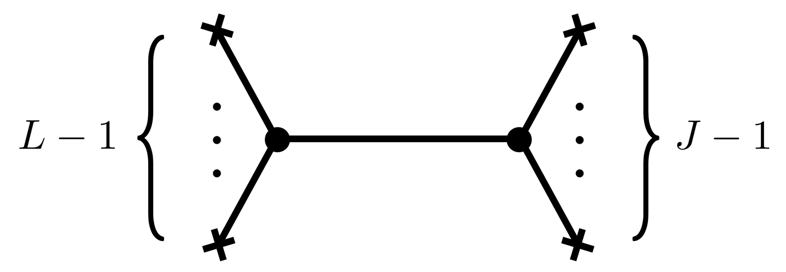
The Two-Point Functions

- Let $Z=\phi_3+i\phi_6$ be a complex scalar field. The propagator is then

$$\langle Z_{\ell m}(x)Z_{\ell' m'}(y)\rangle = (-1)^{m'}\delta_{\ell\ell'}\delta_{m,-m'}\frac{g^2}{Nx_3y_3}\frac{{}_2F_1(\ell,\ell+1;2\ell+2;-\xi^{-1})}{\binom{2\ell+1}{\ell+1}\xi^{\ell}(\xi+1)} \tag{8}$$

where $Z=(Z)_{\ell m}\hat{Y}_\ell^m$ is expanded in fuzzy spherical harmonics.

- The g^2 corrections to the connected part of the bulk two-point functions e.g. $\langle \operatorname{tr} Z^L(x) \operatorname{tr} Z^J(y)\rangle$ are found from diagrams of the form



1 + 2 = 3

- The bulk conformal block decomposition of the two-point functions relates one- and two-point functions of the dCFT to three-point functions of $\mathcal{N}=4$ sYM.
- Comparing two-point functions with perturbative expansions of the bootstrap equations gives relations between different loop-order data, e.g. at tree level

$$\sum_{k_\Delta}\lambda_{ijk}^{(0)}a_k^{(0)} = \text{polynomial expression in } k \tag{9}$$

- For $\langle \operatorname{tr} Z^J \operatorname{tr} X^2\bar{Z}\rangle$ with J odd, at leading N, the operators that contribute are the *BMN* operators. Using the different k-dependence of one-point functions we can disentangle the individual three-point functions $\langle \operatorname{tr} Z^J \operatorname{tr} X^2\bar{Z}\mathcal{O}_{\text{BMN}}\rangle$.

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Further considerations and outlook

- Comparing the two-point functions to the boundary conformal block decomposition similarly produces restrictions on the bulk-boundary two-point functions μ_{ij}.
- Possibility of mining for $\mathcal{N}=4$ data using the one- and two-point functions.
- This dCFT has a string dual thus the two-point functions allow for further tests of the AdS/dCFT correspondence.

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