Yangian Symmetry for Fishnet Feynman Graphs

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4D $\mathcal{N} = 4$ Super Yang-Mills

- **Superconformal symmetry**
- **Planar integrability**
  - Origin: mysterious
- **AdS/CFT correspondence**

$\gamma-$ twisted 4D $\mathcal{N}=4$ Super Yang-Mills
Bi-scalar limit [Gürdoğan, Kazakov '15]

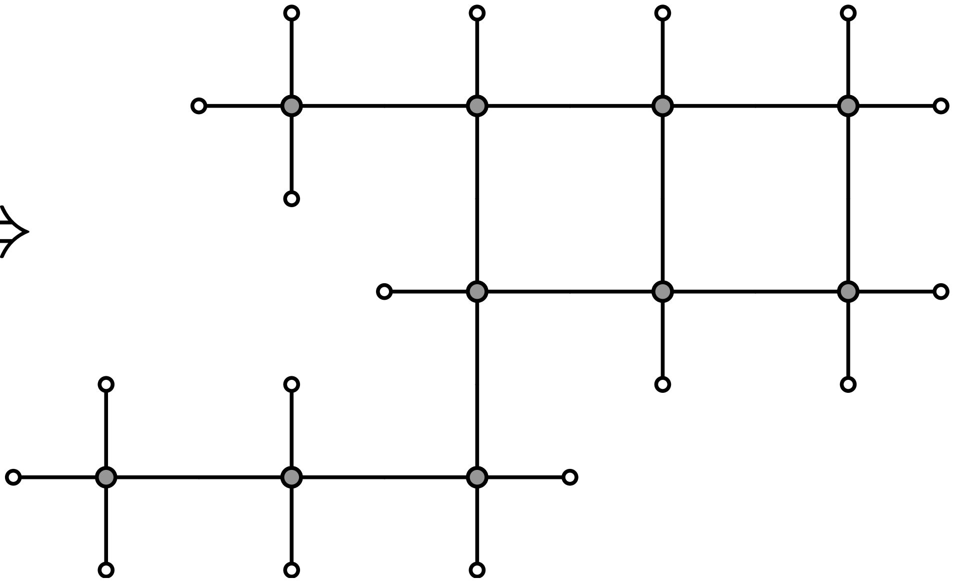
- **Chiral, non-unitary theory**

$$\mathcal{L}_\phi = N_c \mathrm{Tr}\left(\partial^\mu \phi_1^\dagger \partial_\mu \phi_1 + \partial^\mu \phi_2^\dagger \partial_\mu \phi_2 + \xi^2\, \phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2\right)$$

- **1-1 correspondence: correlator and fishnet Feynman diagram**

$$K(x_1,\ldots,x_n) = \langle \mathrm{Tr}[\chi_1(x_1)\ldots\chi_n(x_n)]\rangle. \quad \Leftrightarrow$$

$$\chi_k \in \{\phi_1, \phi_2, \phi_1^\dagger, \phi_2^\dagger\}$$



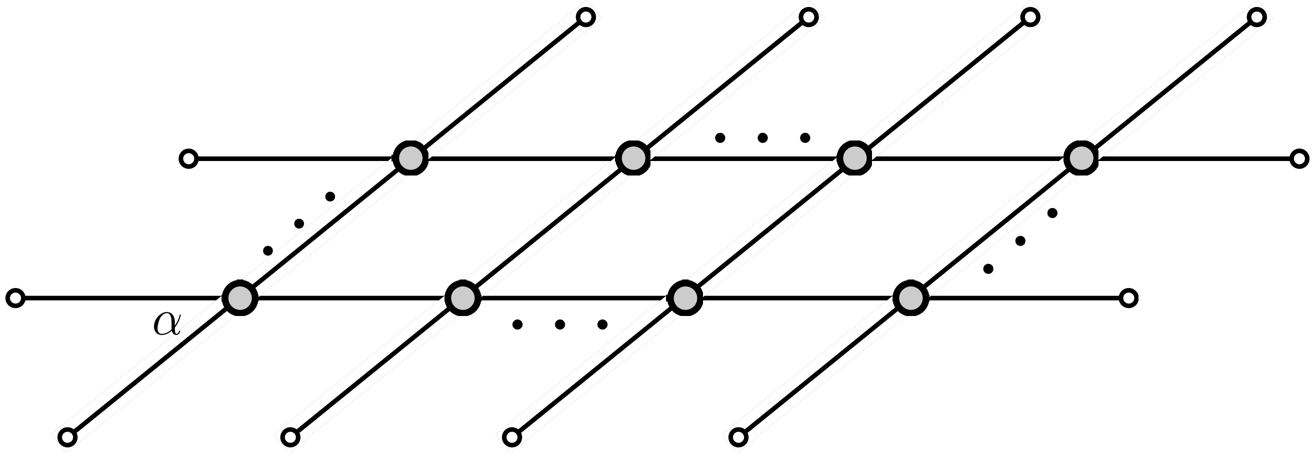
$\gamma -$ twisted 4D $\mathcal{N} = 4$ Super Yang-Mills
Bi-scalar limit [Gürdoğan, Kazakov '15]

- **Conformal symmetry for specific choice of couplings**
- **Planar integrability** [Sieg-Wilhelm '16] [Grabner, Gromov, Kazakov, Korchemsky '17]
  - From first principle
  - Yangian symmetry for **individual Feynman diagrams**
- **Is there an AdS dual?**

Scalar Fishnet Diagrams in 4D

[Zamolodchikov ’80]

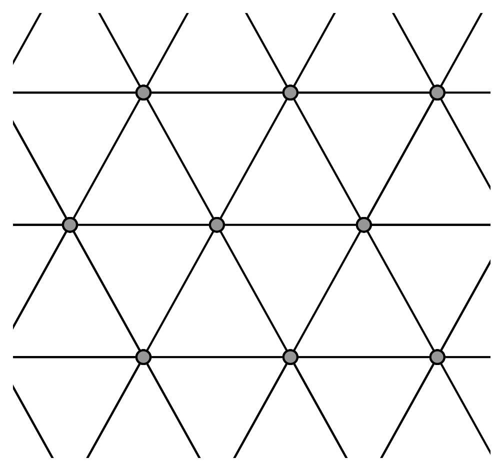
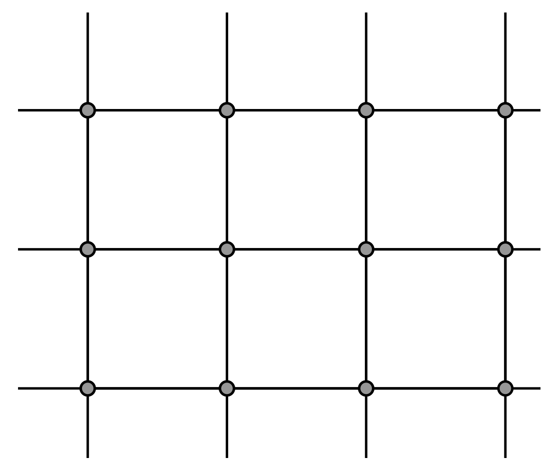
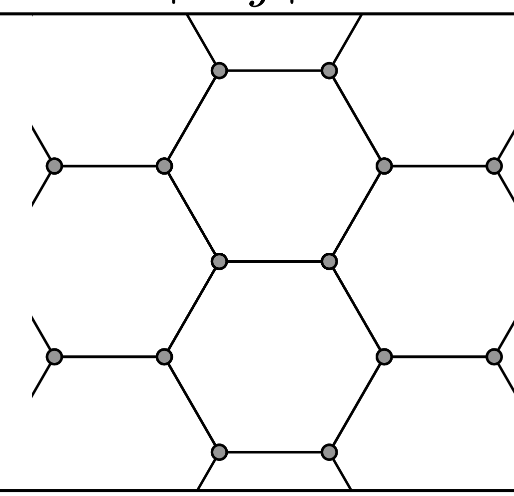
❖ **Starting point: integrable lattice model** [Baxter ’78]



❖ **For** $\alpha = \frac{\pi}{2}$ **:** ϕ^4 **type scalar Feynman diagram**

Scalar Fishnet Diagrams in General

[Zamolodchikov '80]

Dimension	$d = 3$	$d = 4$	$d = 6$
Propagator	$\lvert x_{ij}\rvert^{-1}$	$\lvert x_{ij}\rvert^{-2}$	$\lvert x_{ij}\rvert^{-4}$
Scalar Fishnet			

- **Feynman rules**

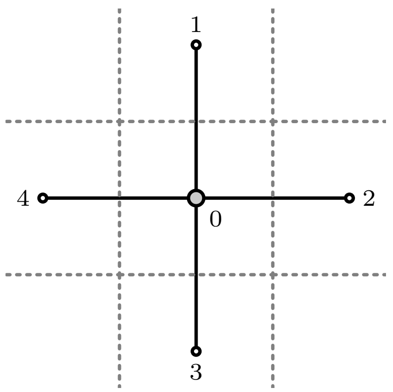
  Vertex $\int d^d x \Leftrightarrow$ ●

  Propagator  $\Leftrightarrow |x_{ij}|^{d-2}$

- **UV/IR finite: unbroken conformal symmetry**

Yangian Symmetry for the Single Box

$$I_4 = \int d^4x_0 \frac{1}{x_{10}^2 x_{20}^2 x_{30}^2 x_{40}^2} \overset{u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}}{\underset{v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}}{=\!=\!=}} \frac{1}{x_{13}^2 x_{24}^2} \Phi(u,v) =$$



❖ **We know everything about this building block!**

- The integral can be expressed in terms of dilogarithms [Ussyukina, Davydychev, '93]
- The integral is **conformally invariant**

$$\mathrm{J}^a I_4 = 0, \quad \mathrm{J}^a = \sum_{j=1}^{4} \mathrm{J}_j^a$$

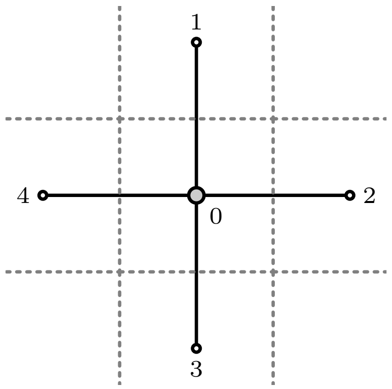
Lorentz Special Conf.

$$\mathrm{J}^a \in \{\mathrm{D}^{\Delta=1}, \mathrm{L}_{\mu\nu}, \mathrm{P}_\mu, \mathrm{K}_\mu\}$$

Dilatation Translation

Yangian Symmetry for the Single Box

$$I_4 = \int d^4x_0 \frac{1}{x_{10}^2 x_{20}^2 x_{30}^2 x_{40}^2} \underset{v=\frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}}{\overset{u=\frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}}{=\!=}} \frac{1}{x_{13}^2 x_{24}^2} \Phi(u,v) =$$



❖ **We know everything about this building block!**

- We also have **non-local hidden symmetry**

$$\widehat{\mathrm{P}}^\mu I_4 = 0, \quad \widehat{\mathrm{P}}^\mu = -\frac{i}{2} \sum_{j<k=1}^{4} \left[(\mathrm{L}_j^{\mu\nu} + \eta^{\mu\nu} \mathrm{D}_j) \mathrm{P}_{k,\nu} - (j \leftrightarrow k) \right] - \sum_{j=1}^{4} j \mathrm{P}_j^\mu$$

Bilocal piece: universal — Depend on the graph

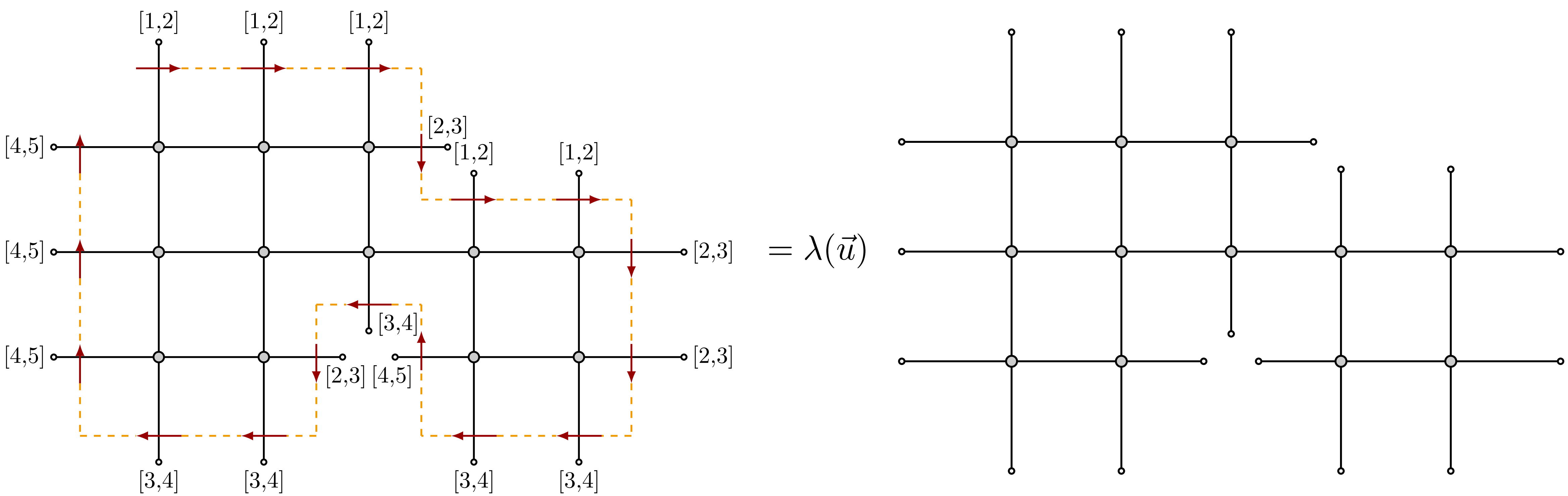
- Hidden symmetry gives two second order **differential equation**

$$0 = \Phi + (3u-1)\frac{\partial \Phi}{\partial u} + 3v\frac{\partial \Phi}{\partial v} + (u-1)u\frac{\partial^2 \Phi}{\partial u^2} + v^2 \frac{\partial^2 \Phi}{\partial v^2} + 2uv \frac{\partial^2 \Phi}{\partial u \partial v}$$

$$0 = \Phi + (u \leftrightarrow v)$$

Yangian Symmetry for Scalar Fishnets

❖ **The general fishnet can be obtained by connecting single boxes**



❖ **The hidden symmetries come from integrability !**

❖ **Fully encoded in the RTT relation**

$$\mathrm{R}_{12}(u-v)\mathrm{T}_1(u)\mathrm{T}_2(v) = \mathrm{T}_2(v)\mathrm{T}_1(u)\mathrm{R}_{12}(u-v)$$

Yangian Symmetry for Scalar Fishnets

❖ **What is R and T?**

- R-matrix: Yang's R-matrix
- T-matrix: product of Lax operator with inhomogeneities

$$\mathrm{T}(\vec{u}) = \mathrm{L}_n[\delta_n^+, \delta_n^-]\mathrm{L}_{n-1}[\delta_{n-1}^+, \delta_{n-1}^-]\cdots \mathrm{L}_1[\delta_1^+, \delta_1^-]$$

- Lax operator: spinorial Lax operator [Chicherin, Derkachov Isaev '13]

Spin generator

$$\mathrm{L}_{k,\alpha\beta}(u_k^+, u_k^-) = u_k\, \mathbb{1}_{k,\alpha\beta} + \tfrac{1}{2}\mathrm{S}_{\alpha\beta}^{ab}\, \mathrm{J}_{k,ab}^{\Delta_k}$$

Inhomogeneities Differential conformal generator

$$u_k^+ := u_k + \frac{\Delta_k - 4}{2}, u_k^- := u_k - \frac{\Delta_k}{2}$$
$$[\delta^+, \delta^-] := (u^+, u^-) \equiv (u + \delta^+, u + \delta^-)$$
$$[\delta] := u + \delta$$

Yangian Symmetry for Scalar Fishnets

❖ **T includes all symmetry generators**

$$\mathrm{T}(u) = u^n\,\mathbb{1} + u^{n-1}\,\mathrm{J} + u^{n-2}\,\widehat{\mathrm{J}} + \cdots$$ Higher level generators

J Level 0 generators (Conformal symmetry)

$\hat{\mathrm{J}}$ Level 1 generators (Dual conformal symmetry)

❖ **Yangian symmetry algebra** [Drinfeld '85]

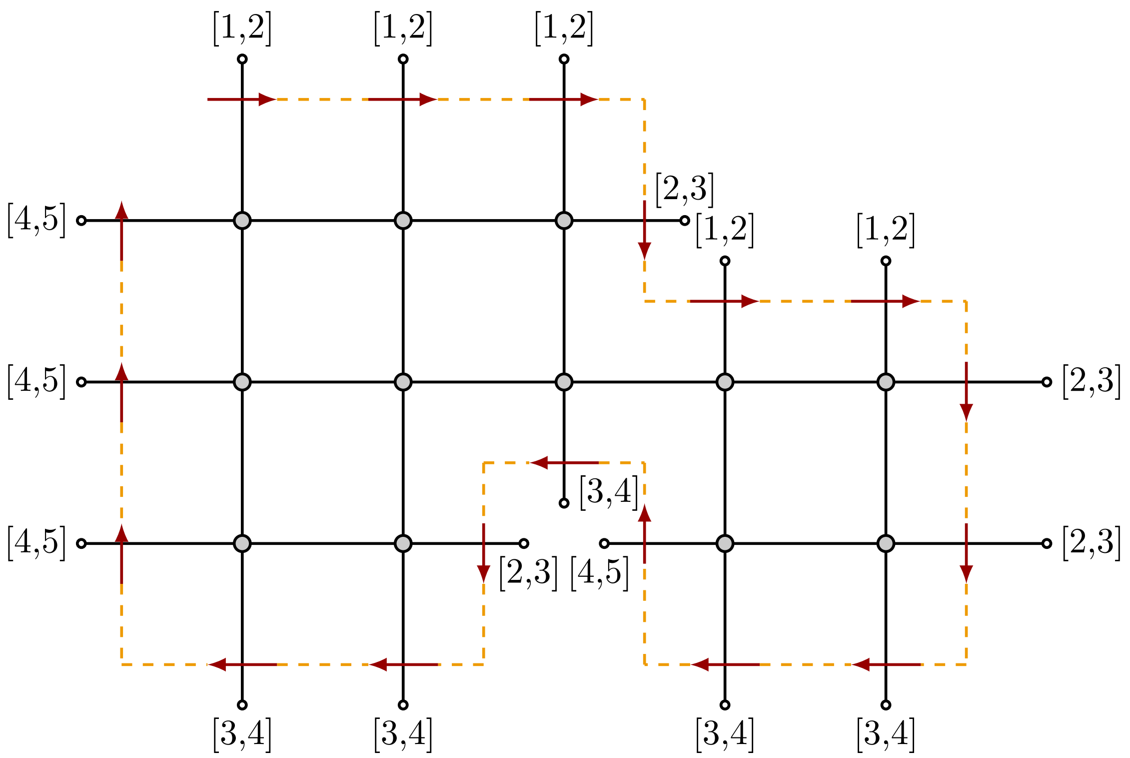
Level 0 $$[\mathrm{J}_a, \mathrm{J}_b] = f_{ab}^{c}\mathrm{J}_c$$

Level 1 $$[\mathrm{J}_a, \hat{\mathrm{J}}_b] = f_{ab}^{c}\hat{\mathrm{J}}_c$$

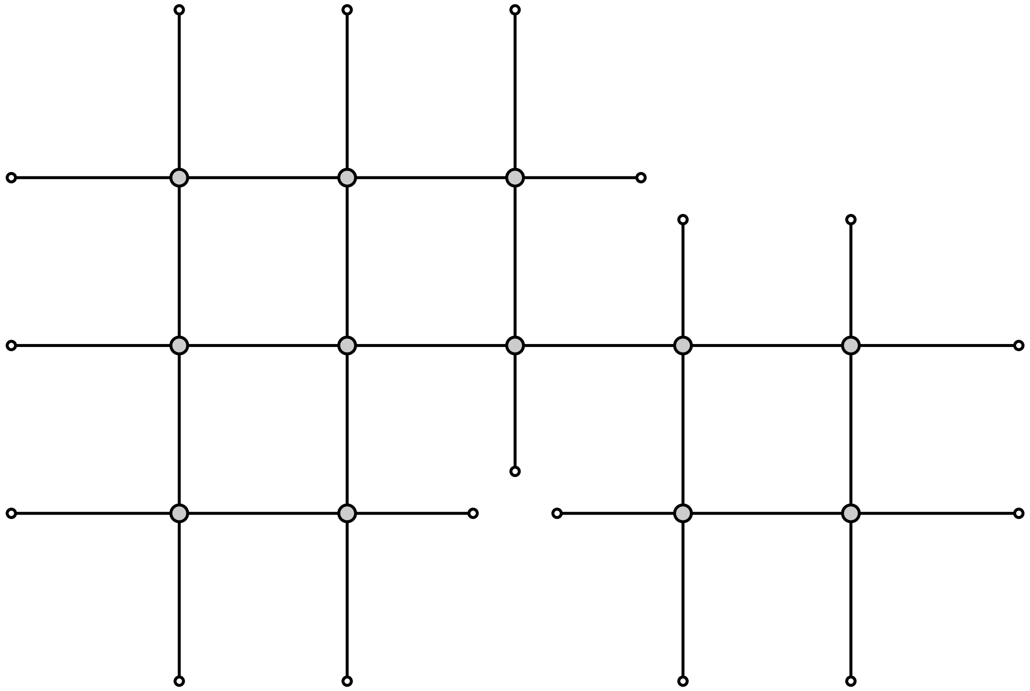
Serre relation $$[\hat{\mathrm{J}}_a, [\hat{\mathrm{J}}_b, \mathrm{J}_c]] - [\mathrm{J}_a, [\hat{\mathrm{J}}_b, \hat{\mathrm{J}}_c]] = \mathcal{O}(\mathrm{J}^3)$$

Yangian Symmetry for Scalar Fishnets

❖ **Yangian symmetry: eigenvalue equation for Feynman integral** I_n



$= \lambda(\vec{u})$

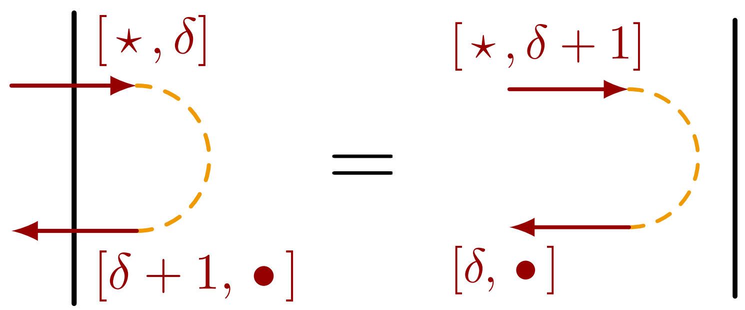


$$\mathrm{T}(\vec{u})\, I_n = \lambda(\vec{u})\, I_n\, \mathbb{1}, \quad \lambda(\vec{u}) = \prod_{j \in \mathrm{out}} \delta_j^+ \delta_j^- = [3]^5 [4]^9 [5]^4 \quad [\delta] := u + \delta$$

Yangian Symmetry for Scalar Fishnets

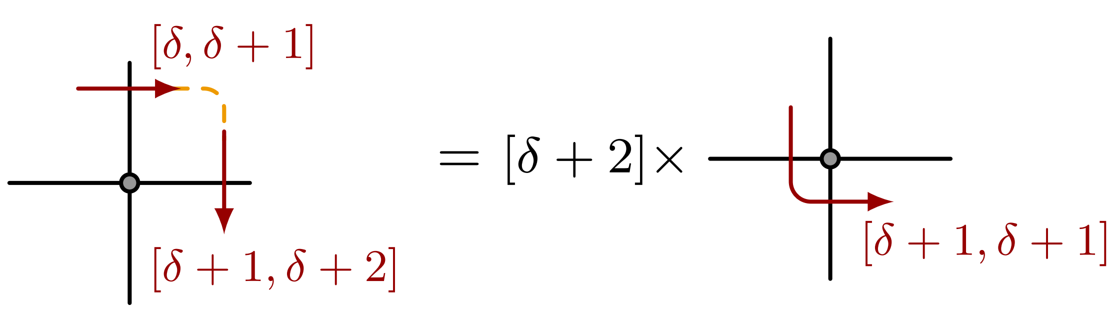
❖ Proof of Yangian symmetry: intertwining relations

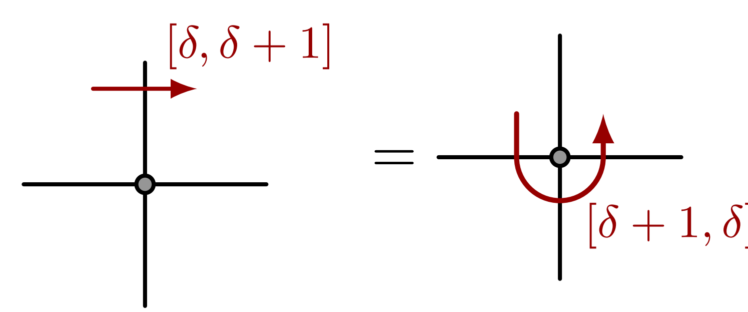
- Intertwiners



$$\Leftrightarrow \quad \frac{1}{x_{12}^2} \mathrm{L}_2[\delta, \bullet] \mathrm{L}_1[\star, \delta+1] = \mathrm{L}_2[\delta+1, \bullet] \mathrm{L}_1[\star, \delta] \frac{1}{x_{12}^2}.$$

- Action on the interaction vertex





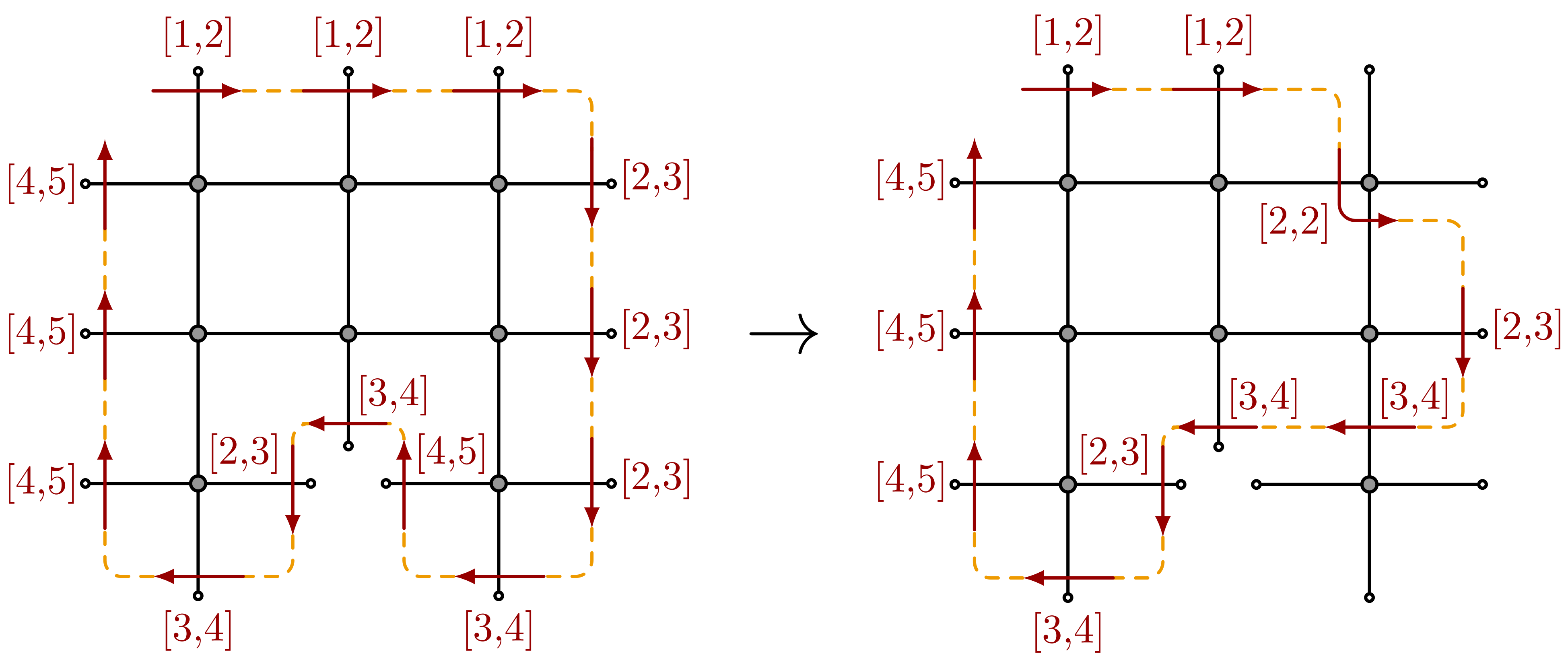
- Vacuum

Integration by parts

$$\mathrm{L}_{\alpha\beta}[\delta, \delta+2] \cdot 1 = \mathrm{L}^T_{\alpha\beta}[\delta+2, \delta] \cdot 1 = [\delta+2]\delta_{\alpha\beta}.$$

Yangian Symmetry for Scalar Fishnets

❖ **Proof of Yangian symmetry: example**



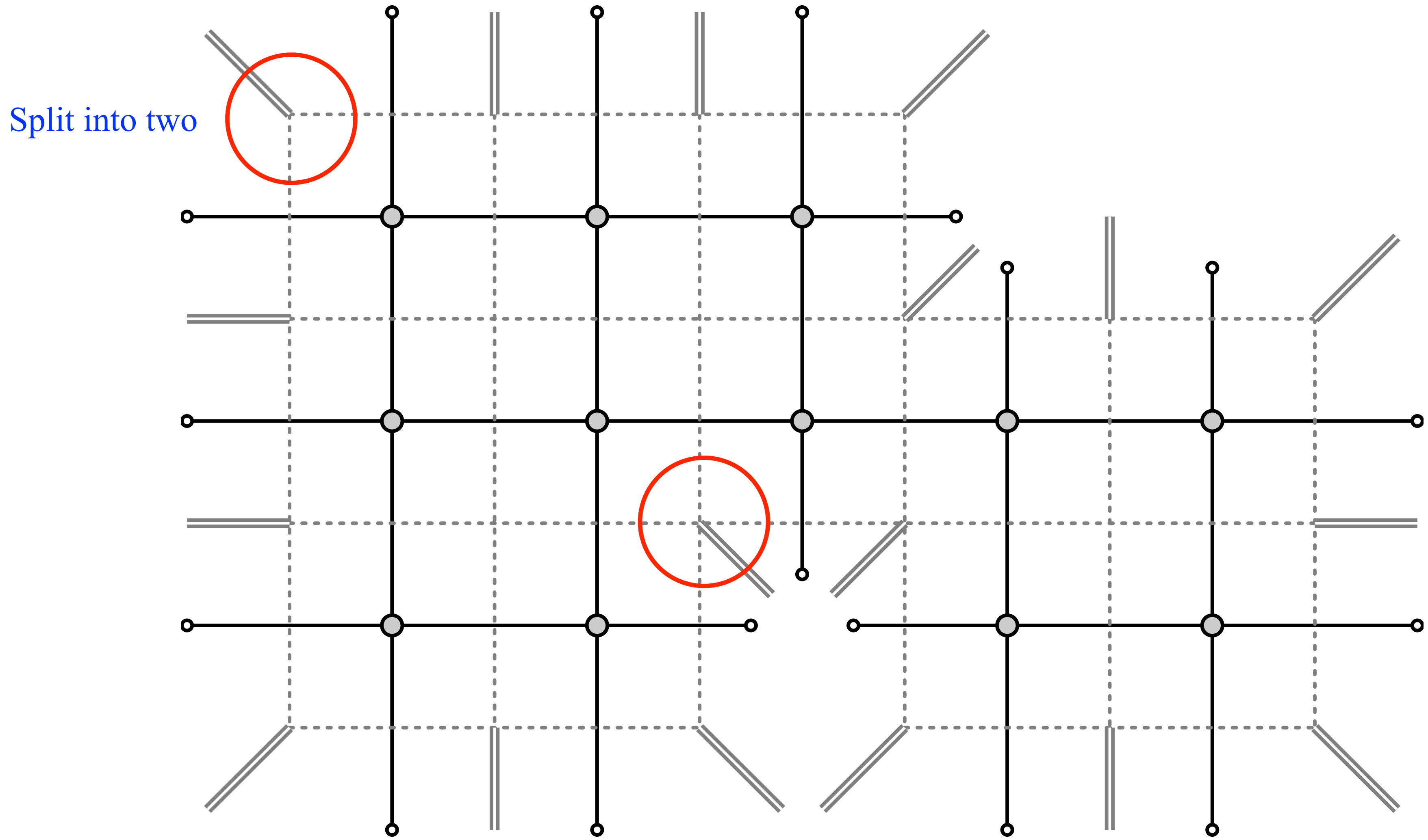
❖ **The end of the story.**

… wait a minute!

Scalar Fishnets in Momentum Space

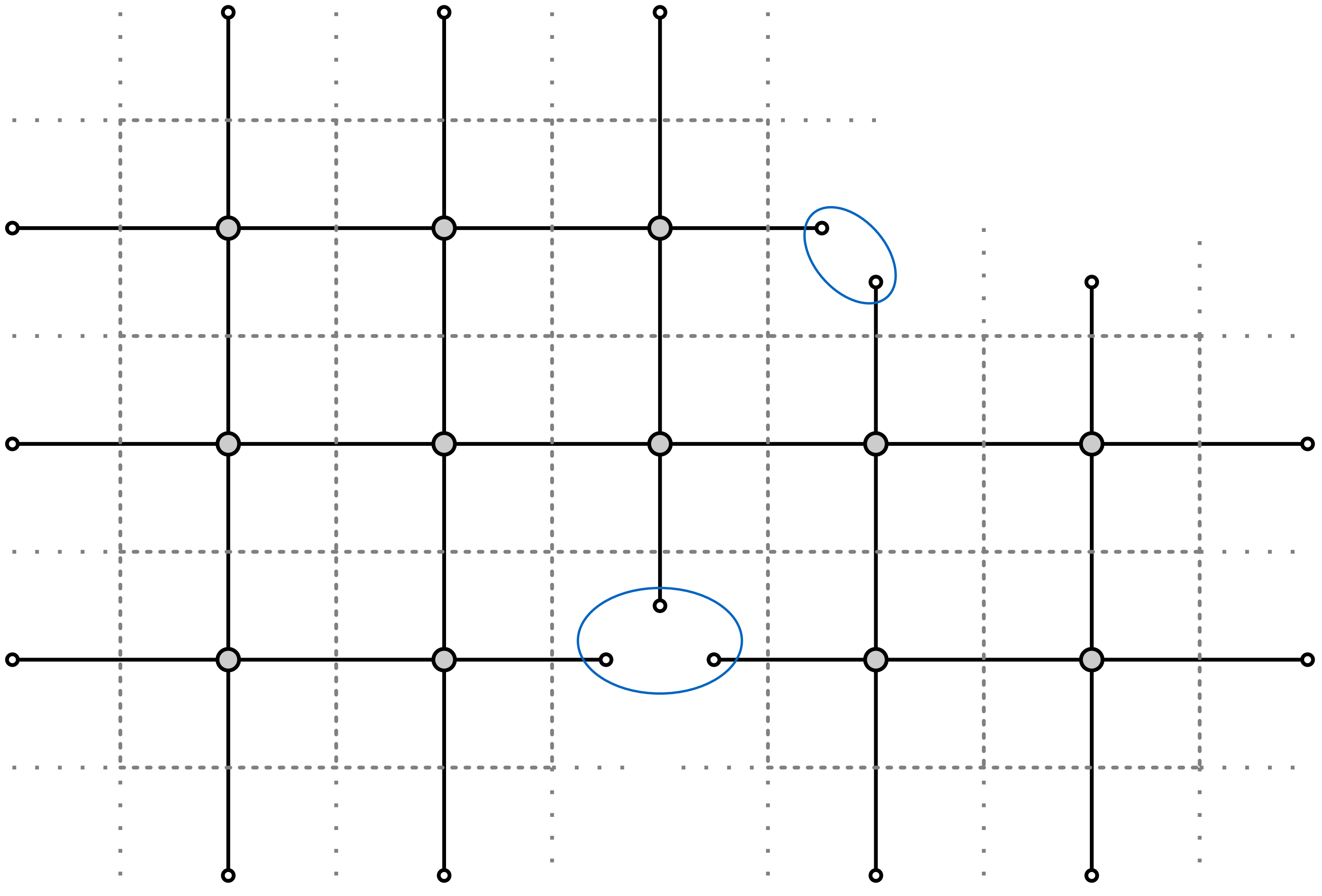
❖ **The dual Feynman diagram in momentum space**

- Problem 1: different kind of interaction vertices



Scalar Fishnets in Momentum Space

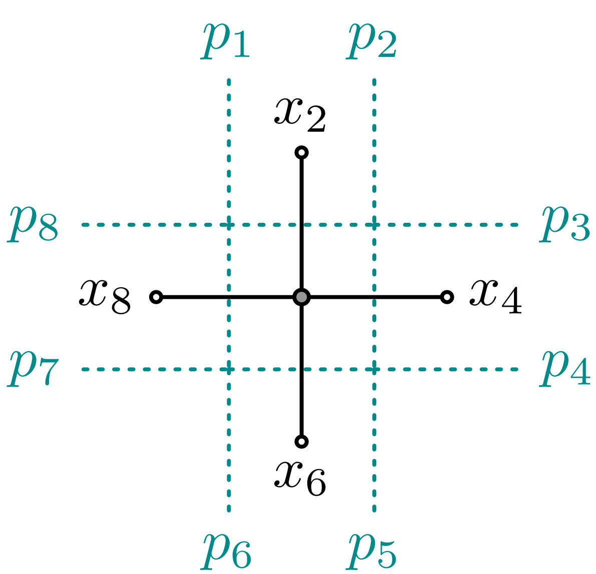
- **The dual Feynman diagram in momentum space**
  - Solution: joining external points



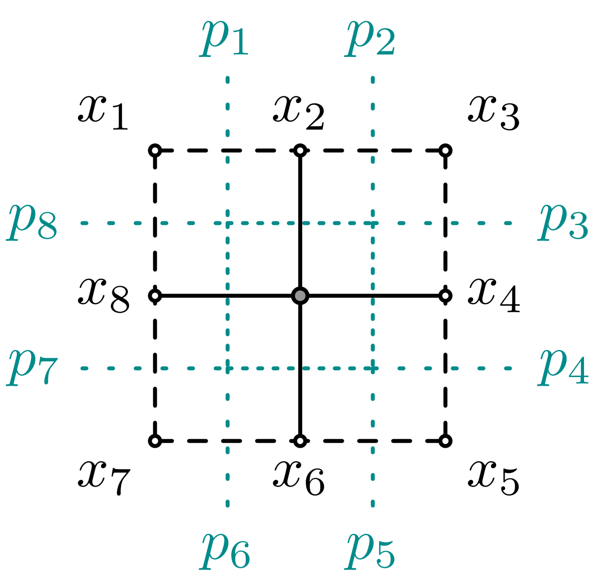
Scalar Fishnets in Momentum Space

❖ **The dual Feynman diagram in momentum space**

- Problem 2: All external legs are off-shell
- Solution: adding delta-functions



Off-shell external legs



On-shell external legs

Scalar Fishnets in Momentum Space

❖ **The dual Feynman diagram in momentum space**

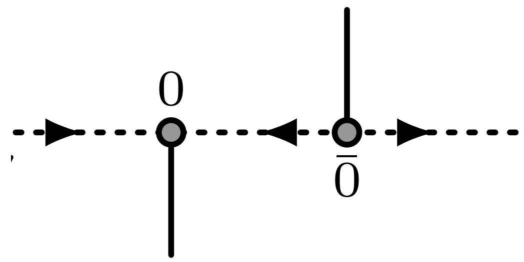
- Direct way of doing: Lax operator in momentum space
- We can obtain it by using Fourier transformation

$$L^{(x)}(u_+, u_-) \int d^D p \, e^{ipx} f(p) \equiv \int d^D p \, e^{ipx} L^{(p)}(u_+, u_-) f(p)$$

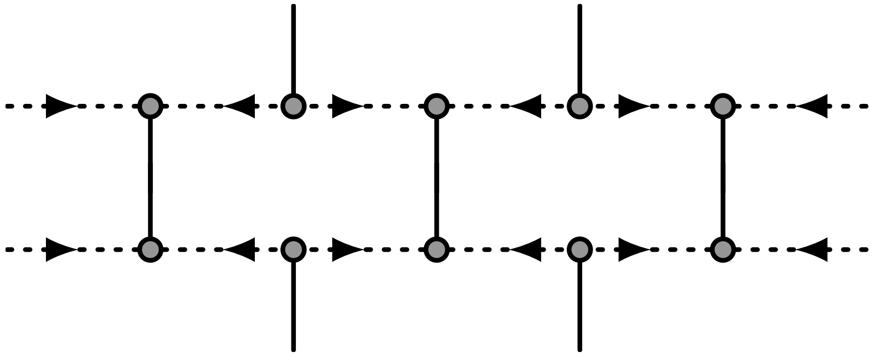
- Similar intertwining relations

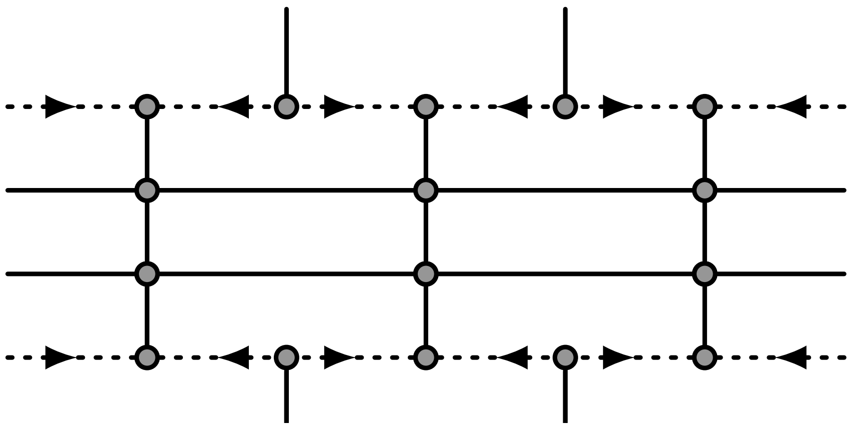
Fishnets with Fermions in 4D

- **New ingredients: Yukawa vertices & fermion line** (dashed line)



- **"Brick wall" diagrams**





- **Lax operator with spin** [Chicherin, Derkachov Isaev '13]

- **Still have Yangian symmetry!**

Summary

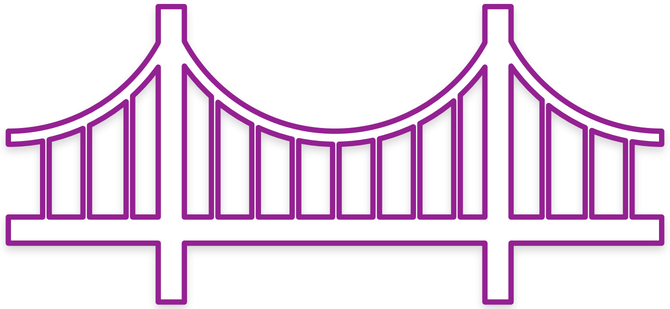
❖ **Yangian symmetries (PDEs) for Feynman diagrams**

❖ **Integrability for non-vanishing dual Coxeter number**

❖ **New kind of fishnet diagrams: brick wall**

Fishnet diagrams

$\mathcal{N} = 4$ SYM



$\gamma -$ twisted $\mathcal{N} = 4$ SYM

Bi-scalar limit

Outlook

- Yangian symmetry for the **full γ-deformed theory** (3 bosons and 3 fermions) ?

- Yangian symmetry in **3d and 6d** for models containing both bosons and fermions?

- For on-shell massless scattering processes, how to understand Yangian symmetry if we have **anomaly**-like behaviour arising from collinear particles? [Chicherin, Sokatchev '17]

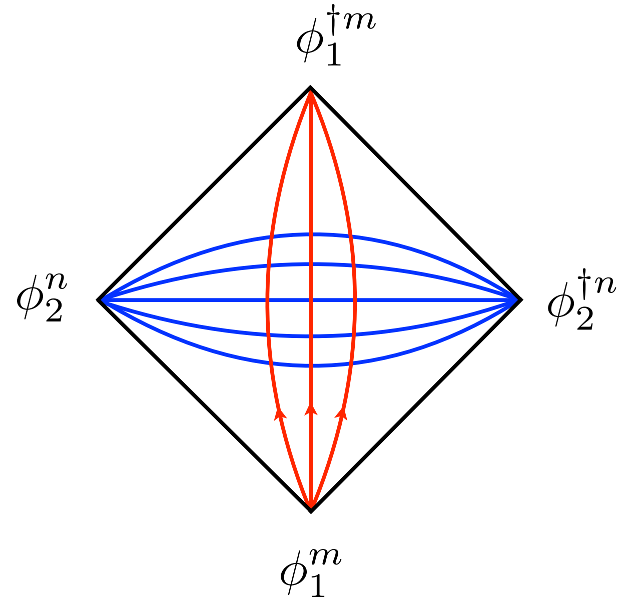
- Using Yangian PDEs to **compute** the Feynman integrals?

- Deriving Yangian symmetry of correlators and amplitudes directly from the Lagrangian ?

Outlook

❖ **For special cases of Fishnet (4pt case), integrability gives integral representations for fishnets.**

[Basso, Dixon '17]



- How to find their Yangian symmetry?
- For m, n large but their ratio fixed, one can solve the saddle point equation exactly.

  [Basso, Dixon, Kosower, D-l.Z 'In progress]
- Continuum limit hints at AdS dual?

Thank you!