4D $\mathcal{N} = 4$ Super Yang-Mills

- Superconformal symmetry
- Planar integrability
  - Origin: mysterious
- Ads/CFT correspondence

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The Integrability Story

![Fishnet diagrams](diagram.png)

- We see that the box integral is indeed Yangian-twisted.

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Scalar Fishnet Diagrams in General

<table>
<thead>
<tr>
<th>Dimension</th>
<th>$d = 3$</th>
<th>$d = 4$</th>
<th>$d = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propagator</td>
<td>$</td>
<td>x_{12}</td>
<td>^2$</td>
</tr>
<tr>
<td>Scalar Fishnet</td>
<td><img src="diagram.png" alt="Diagram" /></td>
<td><img src="diagram.png" alt="Diagram" /></td>
<td><img src="diagram.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>

- Feynman rules
  - Vertex
  - Propagator
- UV/IR finite: unbroken conformal symmetry

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Example: Yangian Symmetry for the Single Box

$$ I_A = \int d^4x_0 \left( \frac{1}{x_{12}^2} \right)^2 \Phi(u, v) = \frac{1}{|x_{12}|^4} \Phi(u, v) $$

- We know everything about this building block!
  - The integral can be expressed in terms of dilogarithms.
  - The integral is conformally invariant.
  - We also have non-local hidden symmetry.

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Yangian Symmetry for Scalar Fishnets

- The general fishnet can be obtained by connecting single box
- The hidden symmetries come from integrability!
- Integrability is fully encoded in the RTT relation
  $$ R_{12}(u-v)T_1(v)T_2(u) = T_2(v)T_1(u)R_{12}(u-v) $$
- $T$ includes all symmetry generators
  $$ T(u) = u^\alpha 1 + u^{\alpha-1} J + u^{\alpha-2} J^2 + \cdots $$
- What is $R$ and $T$?
  - R-matrix is Yang’s R-matrix
  - Transfer matrix: product of Lax operator with inhomogeneities
  - Lax operator is known
  - We can get differential equations from the eigenvalue problem
  $$ T(u) I_A = \lambda(u) I_A \Rightarrow \lambda(u) = \prod_{i<j} \frac{u - u_i}{u - u_j} $$

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Fishnets with Fermions

- New ingredients: Yukawa vertices & fermion line (dashed line)
- "Brick wall" diagrams
- Lax operator with spin

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Problems for the Future

- Can we enhance the Yangian symmetry for the full $\gamma$-deformed theory with 3 bosons and 3 fermions?
- Can we generalise Yangian symmetry to 3d and 6d for models containing both bosons and fermions?
- Can we derive our Yangian symmetry of correlators and amplitudes directly from the Lagrangian?
- For on-shell massless scattering processes, how to understand Yangian symmetry if we have anomaly-like behavior arising from collinear particles?
- Can we solve amplitudes/correlators by using Yangian differential equations? To compare with other approaches, e.g. [Brower-Dixon ’17]
- Etc.