The Isomonodromy Method

The Isomonodromy method describes a family of non-trivial set of Garnier systems with the same monodromy data. This family was first described by Sibuya. All the equations of the same family must have $A_i(t)$ satisfying the following system of equations:

$$\frac{\partial A_i}{\partial t} = \sum_{j=1}^{n} A_j A_{ij} A_{2i}$$

(10)

The system of equations (5) is required from physical grounds to be analytic on the whole plane $\mathbb{C}$; it will have an essential singularity at $z=0$, but analytically at $z=0$, ensures that the quantum state defined by the solution has finite expectation values for the relevant physical quantities (like the bosonic number operator). This condition is translated to our language by requiring that the matrix $C^{(t)}_{\gamma_{12}}$ which connects the natural solutions at $z=0$ and $z=t$ is diagonal: the analytic solution at $z=0$ will also be analytic at $z=t$. In principle the connection could be "upper triangular"; the second solution at $z=0$, which diverges as $z^{\beta_{\gamma}}$, could be connected to a superposition of the divergent and the regular solutions at $z=t$, but one can easily see that this does not happen: consider the determinant of the fundamental matrix, $\det \Phi$, which satisfies the equation

$$\frac{d}{dt} \det \Phi = \left( \frac{2}{\pi} \right) \left( \frac{2}{\pi} \right) \det \Phi.$$

This equation yields $\det \Phi = \Phi_z^2 t^{-\beta_{\gamma}}$ and the result can be used to "normalize" the solutions, i.e. find the constant factor $\chi_{\gamma_{12}}$ in (5). The normalization can be written explicitly as

$$\Phi_{\gamma_{12}} = C^{(t)}_{\gamma_{12}} = \chi_{\gamma_{12}} C^{(0)}_{\gamma_{12}}.$$

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