Charmonium spectroscopy on dynamical anisotropic lattices

K. Jimmy Juge[a], * Alan Ó Cais[a], Mehmet B. Oktay[a], Mike J. Peardon[a], Sinéad M. Ryan[a†], Jon-Ivar Skullerud[a]

[a]School of Mathematics, Trinity College, Dublin 2, Ireland

Preliminary results for the charmonium spectrum on dynamical anisotropic lattices are presented. All-to-all propagators and a variational basis of operators are used to determine the higher-lying states in the spectrum as well as hybrids and exotics.

1. INTRODUCTION

There is a rich spectroscopy of newly observed experimental states and theoretically allowed exotic states in Charmonium. Charm physics has become the focus of the CLEO-c experiment from which extremely precise measurements are expected. High precision lattice calculations in this region of quark mass are now required if accurate predictions are to be made and a meaningful comparison between experiment and theory is to be carried out. Simulations of particles containing charm quarks in lattice QCD have been subject to large uncertainties: effective theories which rely on an expansion in inverse powers of the quark mass are unreliable and a very fine lattice is required for reliable simulations using a simple relativistic action. There are a number of quenched studies with improved actions which attempt to reduce lattice artifacts [1,2] but very little work has been done on dynamical configurations [3].

We present preliminary results using a new dynamical, anisotropic action which has small mass-dependent discretistion errors and is fully relativistic. Using all-to-all propagators [4] we can make full use of the dynamical configurations and easily construct extended operators to extract the radial and orbital excitations of charmonium.

The charmonium spectrum at finite temperature on these lattices is discussed in Ref [5]

*Present address: Department of Physics,Carnegie Mellon University, Pittsburgh, PA 15213, USA
†speaker

2. ANISOTROPIC ACTION

In this section we describe the gauge and quark actions used in our study of charmonium. The combination has leading discretisation errors of $\mathcal{O}(a_s^4, a_t, \alpha a_s^2)$. No improvement terms are applied in the temporal direction but it is hoped that the fine lattice in this direction ($a_t^{-1} \sim 8$ GeV) makes such terms unnecessary. This will be investigated in future work.

The gauge action used in this study is

$$S_g = \frac{\beta}{\xi_g}\left\{\frac{5(1+\omega)}{3u_s^4}\Omega_s - \frac{5\omega}{3u_s^8}\Omega_s^{(2t)} - \frac{1}{12u_s^6}\Omega_s^{(R)}\right\} + \beta\xi_g\left\{\frac{4}{3u_s^2u_t^2}\Omega_t - \frac{1}{12u_s^4u_t^2}\Omega_t^{(R)}\right\}, \qquad (1)$$

where Ω_s and Ω_t are spatial and temporal plaquettes respectively. $\Omega_s^{(R)}$ and $\Omega_t^{(R)}$ are space-space and space-time rectangles and $\Omega_s^{(2t)} = 1/2\sum_{x,i>j} 1 - P_{ij}(x)P_{ij}(x+\hat{t})$. This action has previously been used in a precision study of glueballs [6].

The quark action [7] which uses stout links [8] is as follows

$$S_q = \bar{\psi}\left(\gamma_0\nabla_0 + \sum_i \mu_i\nabla_i\left(1 - \frac{1}{\xi_q a_s^2}\Delta_i\right) - \frac{ra_t}{2}\Delta_{i0} + sa_s^3\sum_i\Delta_i^2 + m_0\right)\psi. \qquad (2)$$

The same quark action is used to simulate the light sea quarks and heavy valence quarks. In this study the sea quark mass was close to the strange quark mass. We have seen no indications of difficulties simulating at lighter sea quark masses and this is will be investigated in future work.

The ratio of scales, $\xi = a_s/a_t$ appears as a bare parameter in both the gauge and quark actions. ξ_g in Eq. 1 and ξ_q in Eq. 2 must be tuned so that the anisotropy determined from a physical probe takes its target value (six in these simulations). In the quenched approximation this tuning can be done separately for the gauge and quark actions and is not a significant overhead. However, in the dynamical theory it must be carried out simultaneously for the gauge and quark actions, requiring a minimum of three initial tuning runs to determine the correct bare values of ξ_g and ξ_q. This procedure has been described in detail in Ref [9]. The results presented here are determined for values of ξ_q and ξ_g close to the tuned point. A more comprehensive study of charmonium, at the correctly tuned values, is underway.

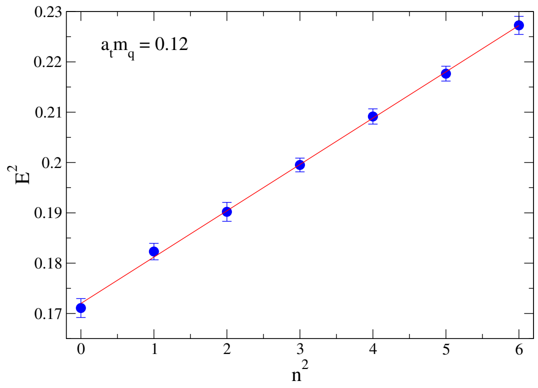

Figure 1. The pseudoscalar energy momentum dispersion relation for a heavy quark of $a_t m_q = 0.12$, heavier than charm. The momentum is plotted for values of n^2 up to six, where $p_n = 2\pi n/La_s$.

3. SIMULATION DETAILS

In this study we have data at two values of the heavy quark mass, $a_t m_q = 0.1$ and $a_t m_q = 0.12$, which straddle the charm quark. The energy-momentum dispersion relations at the quark masses used in this study are relativistic. Figure 1 shows the energy-momentum dispersion relation for the η_c with $a_t m_q = 0.12$. We find impressive linear behaviour for all momenta used in the simulation. The highest momenta corresponds to approximately 1.5 GeV on these lattices. The J/ψ exhibits similar linear behaviour.

At this preliminary stage in our analysis the data sets are not complete and have different numbers of configurations and different dilutions. The details are described in Table 1.

To maximise the overlap with the higher excited states of charmonium we use a variational basis of operators including operators extended in space. The all-to-all propagators are a crucial element in this analysis, allowing us to construct and analyse easily this compliated basis of operators. In particular, we use the "dilution" method of Ref [4] without eigenvectors for the charm quark propagators. As described in Table 1 we have results from time dilution and "time+colour+space-even-odd" dilution. Preliminary investigations indicate that even higher dilutions (eg. including spin) may reduce the errors still further. This is work in progress.

The time-dilution in all-to-all propagators introduces a random noise source on each timeslice which makes locally measured quantities, such as the effective mass, fluctuate more than with point propagators. This is illustrated in Figure 2. This effect can make it difficult to identify a convincing plateau from looking at effective mass plots alone. These local fluctuations do not, however, affect the result of fits to the data since these capture the long-range exponential decay of the correlators. A better picture of the quality of the data is obtained from a sliding-window plot. t_{max} is fixed and fitted values of the mass are plotted against the minimum timeslice used in the fit. A sliding window plot is shown in Figure 3.

The operators used in this study are summarised in Table 2 and are similar to those used in Ref [10]. In addition to the operators, two different smearings of the quark fields were used to increase the size of the variational basis. The optimisation was performed at the largest timeslice possible once the results are independent of this

Table 1
The simulation details for the two bare quark masses, $a_t m_q = 0.1, 0.12$.

Configurations	200 ($am_q = 0.1$)
	50 ($am_q = 0.12$)
Dilution	time ($am_q = 0.1$)
	time,space (even/odd)
	and colour ($am_q = 0.12$)
Physics	radial excitations, hybrids,
	exotics ($am_q = 0.1$)
	S,P,D waves ($am_q = 0.12$)
Volume	$8^3 \times 48$
N_f	2
a_s	~ 0.2fm
a_t^{-1}	~ 8GeV

choice. The metric timeslice is fixed at $t = 1$. Note that operators are included for the hybrid

Table 2
The basis of operators used to project out various higher lying states. The notation for the gluonic paths is that used in Ref. [10]

Particle	Operator
1^{--}	$\vec{\gamma}, \gamma_5\vec{u}, \gamma_5 \times \vec{B}$
0^{-+}	$\gamma_5, \gamma_5(s_1 + s_2 + s_3)$
0^{++}	$\vec{\gamma} \cdot \vec{p}$
1^{++}	$\vec{\gamma} \times \vec{p}$
2^{++}	$\gamma_k p_i + \gamma_i p_k$
1^{+-}	$\gamma_5\vec{p}$
1^{-+}	$\vec{\gamma} \times \vec{u}, \vec{\gamma} \times \vec{B}$
2^{-+}	$\gamma_5(s_1 - s_3), \gamma_5(2s_3 - s_1 - s_3)$
3^{--}	$\vec{\gamma} \cdot \vec{t}$

1^{--} channel and the exotic 1^{-+} channel. For the latter both a staple and a chromomagnetic field were used since the staple is known to have a large overlap with the static $q\bar{q}$ system [11].

4. RESULTS

In the following results the temporal lattice spacing is set from the spin-averaged S-P splitting in charmonium. Using stout links the inverse lattice spacing is $a_t^{-1} = 8.06(7)$GeV. The use of all-to-all propagators on anisotropic lattices results in a dramatic improvement in the signals of the higher charmonium states. Figure 4 shows the sliding window plot for the J/ψ ground state (1S) and first excited state (2S) for a bare quark mass, $a_t m_q = 0.1$.

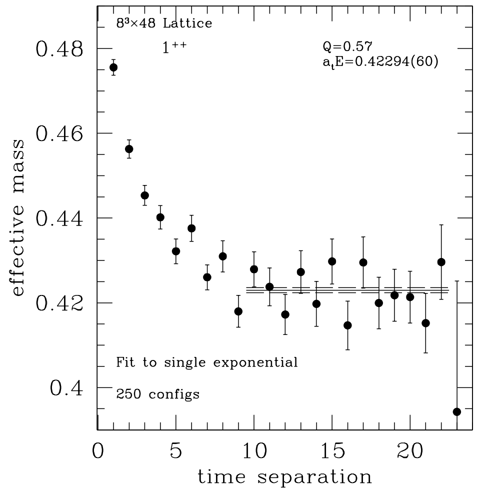

Figure 2. The effective mass for the 1^{++} ground state at zero momentum. The fitted value of the mass and Q-value of the fit are shown in the legend.

The quality of the data shown in this plot is impressive. The errors on the fitted masses of both states, obtained from single exponential fits, is at the 2% level and good fits are found for at least ten different timeslice ranges. The dotted line on the plot indicates the experimentally observed value and not unexpectedly the lattice data lie below since the quark mass used in this fit is lower than the charm quark mass.

At the moment the experimentally observed hyperfine splitting is not seen with this data. At this point the J/ψ and η_c are degenerate within errors. There are however, several reasons which may explain why this splitting is not yet observed

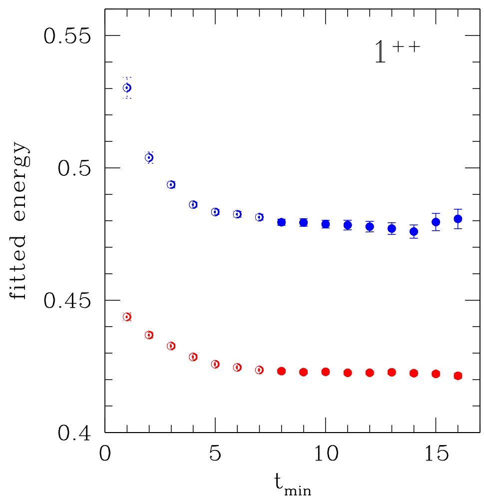

Figure 3. The sliding window plot for the ground state and first excited state of the 1^{++} meson at zero momentum. The closed symbols indicate acceptable fits while the open symbols indicate an unaccepatble fit.

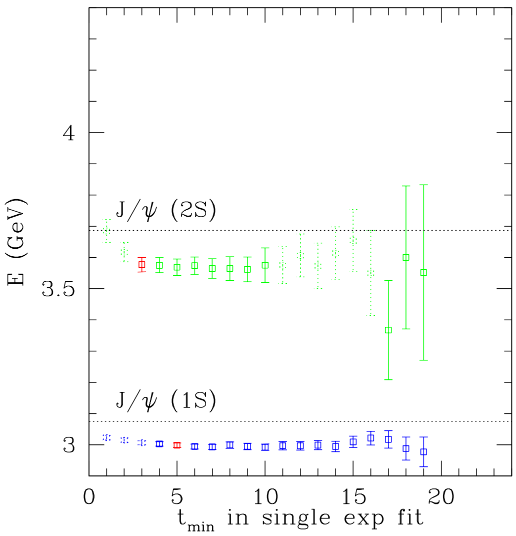

Figure 4. Sliding window plot for the J/ψ 1S and 2S states for $a_t m_q = 0.1$ (lighter than the charm quark mass). The masses are determined from single exponential fits to the data and the preferred t_{min} is highlighted in red. The dotted lines are the experimentally measured particle masses.

including finite volume effects, discretisation errors and light sea quark effects. These have discussed in more detail in Ref [12].

The $c\bar{c}$ spectrum determined from this analysis is shown in Figure 5. The plot shows that the higher excitations of the charmonium spectrum, in particular the D-wave states, can be precisely determined. Since the quark masses have not been tuned to the charm quark mass the spectrum is presented with the J/ψ adjusted to its physical value.

5. CONCLUSIONS

We have presented preliminary results from a simulation of charmonium on a dynamical anisotropic lattice. All-to-all propagators and a variational basis of operators have been used allowing a determination of the spectrum of S, P and D waves and their radial excitations, and the hybrid and exotic states. These preliminary results indicate that precision results can be achieved for the charmonium spectrum. In particular a good signal for D waves and exotic hybrid states is very encouraging. A more detailed study is underway and will address a number of uncertainties. The quark mass must be tuned to the charm quark mass. The bare anisotropies must be correctly tuned so that the target anisotropy, $\xi = 6$ is determined from both the gauge and quark sectors. The contribution of disconnected diagrams has not been evaluated and although this is expected to be small should nevertheless be investigated. Using all-to-all propagators this is a reasonably straighforward task and will be included in future studies. We are currently investigating the finite volume effects by repeating this study on a larger lattice.

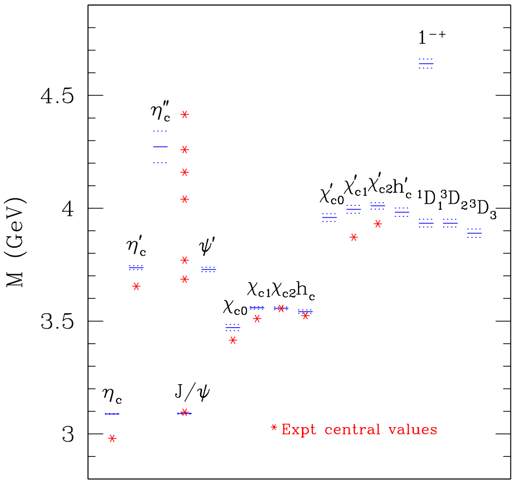

Figure 5. The $c\bar{c}$ spectrum. The scale has been set from the $S-P$ splitting and the J/ψ mass has been adjusted to its physical value to compensate for not precisely tuning the quark mass.

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