MSc Energy (2019/20)

NUCLEAR REACTOR TECHNOLOGY (LECTURE 4)

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Reactor Power and Fuel Consumption (Steady-state Reactor Operation)

- In steady state the reaction probability (rate) is:

$$R = N\Phi\sigma = \frac{M}{A} N_A \Phi\sigma$$

$$\sigma_f = 579\text{b}$$

- The power output is then:

$$P = RE$$

$$E_f \approx 200\text{MeV}$$

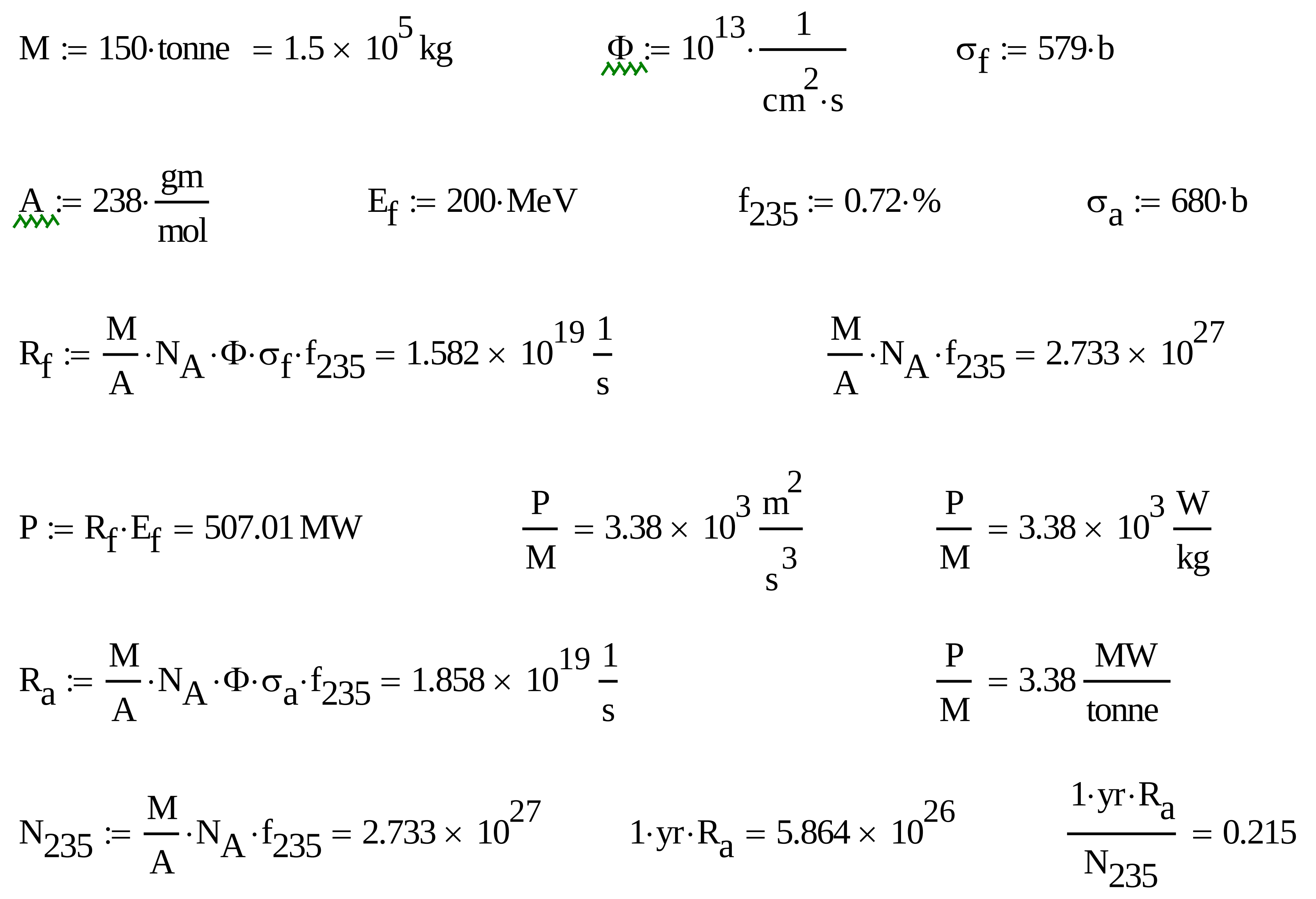
- For fission of ^{235}U

$$P_f = R_f E_f = \frac{M}{A} N_A \Phi \sigma_f E_f f_{235}$$

$$f_{235} = 0.72\%$$

$$A = 238\text{kg/kmol}$$

Steady State Rate and Power Output



$M := 150 \cdot \text{tonne} = 1.5 \times 10^{5}\,\text{kg}$ $\quad \Phi := 10^{13} \cdot \frac{1}{\text{cm}^{2} \cdot \text{s}}$ $\quad \sigma_f := 579 \cdot \text{b}$

$A := 238 \cdot \frac{\text{gm}}{\text{mol}}$ $\quad E_f := 200 \cdot \text{MeV}$ $\quad f_{235} := 0.72 \cdot \%$ $\quad \sigma_a := 680 \cdot \text{b}$

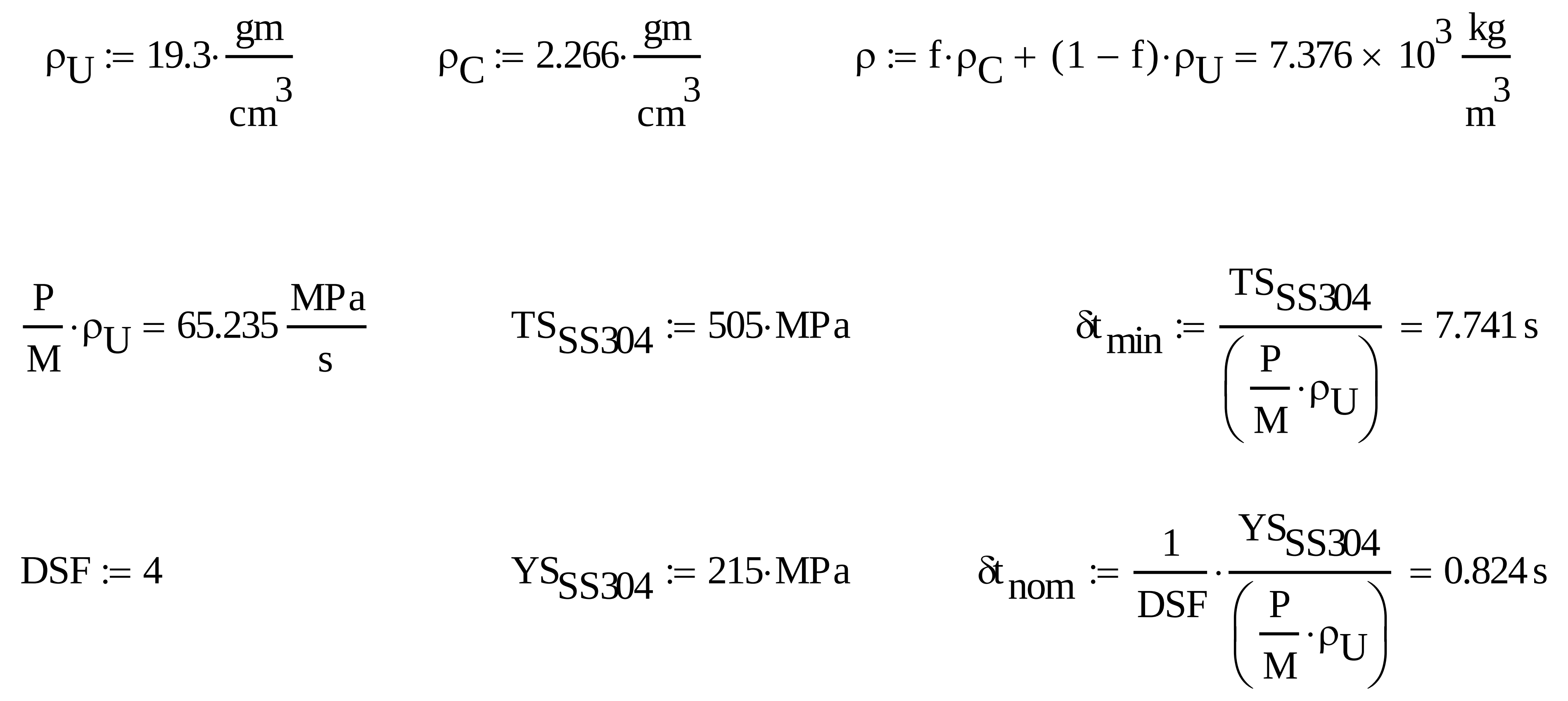
$R_f := \frac{M}{A} \cdot N_A \cdot \Phi \cdot \sigma_f \cdot f_{235} = 1.582 \times 10^{19}\,\frac{1}{\text{s}}$ $\quad \frac{M}{A} \cdot N_A \cdot f_{235} = 2.733 \times 10^{27}$

$P := R_f \cdot E_f = 507.01\,\text{MW}$ $\quad \frac{P}{M} = 3.38 \times 10^{3}\,\frac{\text{m}^{2}}{\text{s}^{3}}$ $\quad \frac{P}{M} = 3.38 \times 10^{3}\,\frac{\text{W}}{\text{kg}}$

$R_a := \frac{M}{A} \cdot N_A \cdot \Phi \cdot \sigma_a \cdot f_{235} = 1.858 \times 10^{19}\,\frac{1}{\text{s}}$ $\quad \frac{P}{M} = 3.38\,\frac{\text{MW}}{\text{tonne}}$

$N_{235} := \frac{M}{A} \cdot N_A \cdot f_{235} = 2.733 \times 10^{27}$ $\quad 1 \cdot \text{yr} \cdot R_a = 5.864 \times 10^{26}$ $\quad \frac{1 \cdot \text{yr} \cdot R_a}{N_{235}} = 0.215$

Characteristic (critical) Mechanical Time Constant



$\rho_U := 19.3\cdot\frac{gm}{cm^3}$

$\rho_C := 2.266\cdot\frac{gm}{cm^3}$

$\rho := f\cdot\rho_C + (1-f)\cdot\rho_U = 7.376\times 10^3\,\frac{kg}{m^3}$

$\frac{P}{M}\cdot\rho_U = 65.235\,\frac{MPa}{s}$

$TS_{SS304} := 505\cdot MPa$

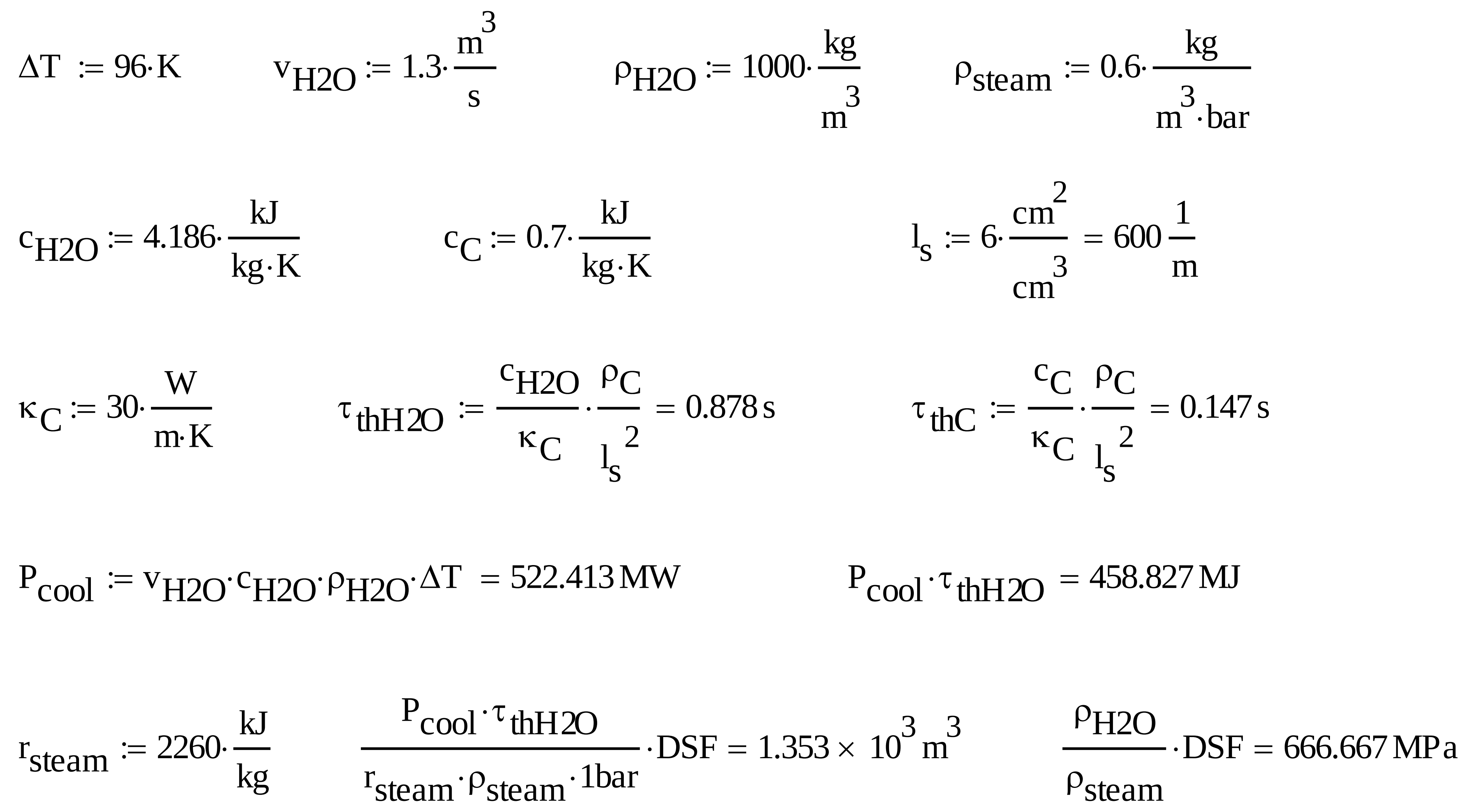
$\delta t_{min} := \frac{TS_{SS304}}{\left(\frac{P}{M}\cdot\rho_U\right)} = 7.741\,s$

$DSF := 4$

$YS_{SS304} := 215\cdot MPa$

$\delta t_{nom} := \frac{1}{DSF}\cdot\frac{YS_{SS304}}{\left(\frac{P}{M}\cdot\rho_U\right)} = 0.824\,s$

Characteristic Thermal Time Constant



$\Delta T := 96\cdot K$

$v_{H2O} := 1.3\cdot\frac{m^3}{s}$

$\rho_{H2O} := 1000\cdot\frac{kg}{m^3}$

$\rho_{steam} := 0.6\cdot\frac{kg}{m^3\cdot bar}$

$c_{H2O} := 4.186\cdot\frac{kJ}{kg\cdot K}$

$c_C := 0.7\cdot\frac{kJ}{kg\cdot K}$

$l_s := 6\cdot\frac{cm^2}{cm^3} = 600\,\frac{1}{m}$

$\kappa_C := 30\cdot\frac{W}{m\cdot K}$

$\tau_{thH2O} := \frac{c_{H2O}}{\kappa_C}\cdot\frac{\rho_C}{l_s^2} = 0.878\,s$

$\tau_{thC} := \frac{c_C}{\kappa_C}\cdot\frac{\rho_C}{l_s^2} = 0.147\,s$

$P_{cool} := v_{H2O}\cdot c_{H2O}\cdot\rho_{H2O}\cdot\Delta T = 522.413\,MW$

$P_{cool}\cdot\tau_{thH2O} = 458.827\,MJ$

$r_{steam} := 2260\cdot\frac{kJ}{kg}$

$\frac{P_{cool}\cdot\tau_{thH2O}}{r_{steam}\cdot\rho_{steam}\cdot 1bar}\cdot DSF = 1.353\times 10^3\,m^3$

$\frac{\rho_{H2O}}{\rho_{steam}}\cdot DSF = 666.667\,MPa$

Time constant for power growth/decay (τ_{pow})

- No reactor can ever operate at a perfectly static, stable power level.
- If only prompt neutrons in the reactor:
  - power levels cannot be 'balanced' .
  - the time constant for power growth/decay is $\tau_{pow} < 1$s.
- Prompt + delayed neutrons → τ_{pow} > minutes or hours.
- For an unmoderated, pure ^{235}U assembly τ_{pow} can be as low as 10^{-8} s.
- This is a fission weapon. Over 10^{11} fissions are produced in 1 μs, most of them in the first 20 ns.
- More fissions occur before the fragments have stopped, the assembly disrupts explosively.

Reactor Kinetics

- Over-critical reactor: $q = k - 1 > 1$

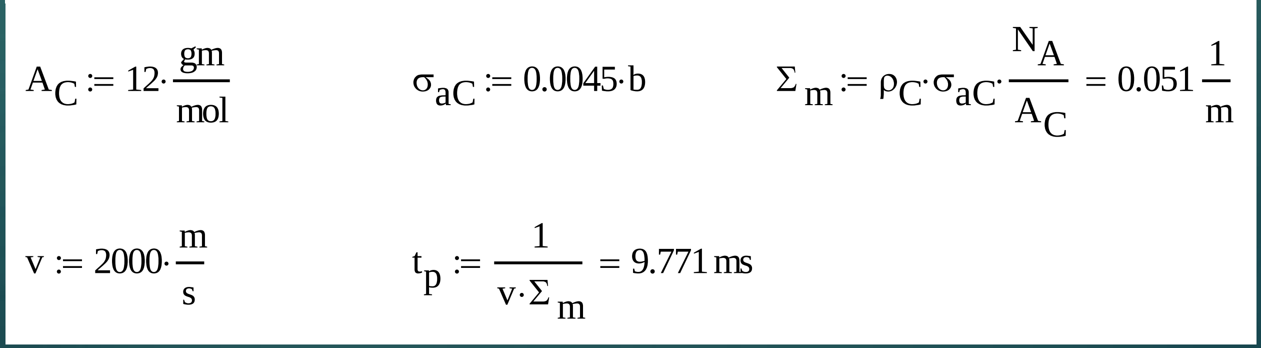
$$n(t) = n_0 \exp(qt / t_p)$$

- For most reactors the prompt time is dominated by diffusion:

$$t_p \approx t_d = \lambda_a / v$$

- Using a moderator dominated reactor limit:

$$t_p \approx 1 / v\Sigma_a(\mathrm{C}) = (1 - f) / v\Sigma_a(M)$$



$A_C := 12\cdot\frac{\mathrm{gm}}{\mathrm{mol}}$

$\sigma_{aC} := 0.0045\cdot\mathrm{b}$

$\Sigma_m := \rho_C\cdot\sigma_{aC}\cdot\frac{N_A}{A_C} = 0.051\,\frac{1}{\mathrm{m}}$

$v := 2000\cdot\frac{\mathrm{m}}{\mathrm{s}}$

$t_p := \frac{1}{v\cdot\Sigma_m} = 9.771\,\mathrm{ms}$

A more complete dynamic equation

- With *d* being the fraction of delayed neutrons:

$$n(t) = n_0 \left\{ \frac{d}{d-q} \exp\left[\frac{qt}{(d-q)\tau_\mathrm{d}} \right] - \frac{q}{d-q} \exp\left[\frac{-(d-q)t}{t_\mathrm{p}} \right] \right\}$$

- In the delayed limit this becomes:

$$d \simeq 0.65\%$$

$$n(t) \sim n_0 \exp\left[qt/(d-q)\tau_\mathrm{d} \right]$$

$$\tau_\mathrm{d} \sim 12.5\mathrm{s}$$

Fission product build-up and reactor poisoning

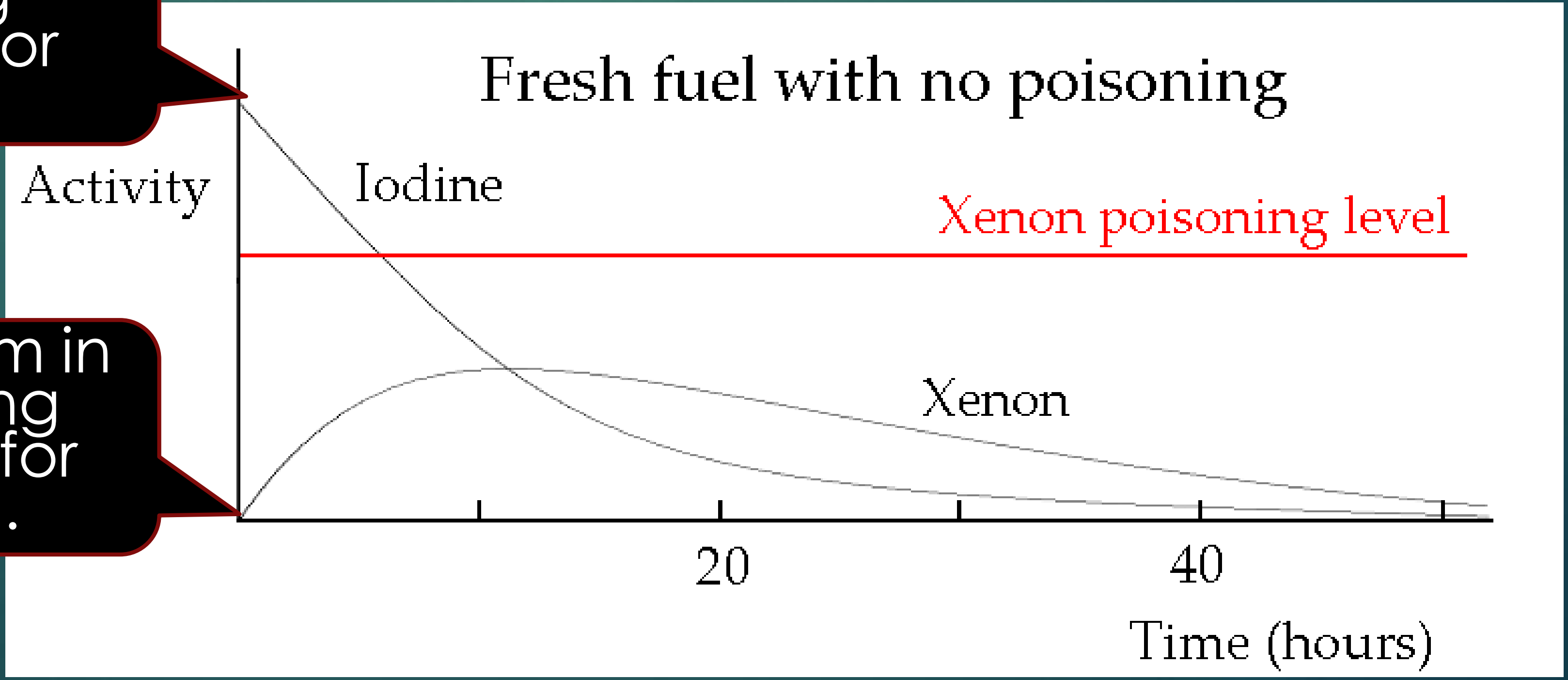
- Fission fragments accumulate in fuel during reactor operation. Many absorb neutrons, so the control rods are withdrawn further to offset this.
- A significant fission fragment decay is:

$$\boxed{{}^{135}_{53}\mathrm{I}\ (t_{1/2} = 6.7\,\mathrm{hr}) \rightarrow {}^{135}_{54}\mathrm{Xe}\ (t_{1/2} = 9.2\,\mathrm{hr}) \rightarrow {}^{135}_{55}\mathrm{Cs}}$$

- This is a parent-daughter growth-decay situation.
- Thermal neutron capture cross section: $\sigma\left({}^{135}_{54}\mathrm{Xe}\right) \approx 10^{6}\ \mathrm{b}$
- If a running reactor with 'old' fuel is slowed or shut down, the ^{135}Xe builds up so much that the reactor cannot be restarted until it decays again in ~ 1 day.
- This is called ***reactor poisoning***.

Reactor poisoning

- If a reactor is shut down at *t* = 0, the activity of ^{135}Xe starts from near zero (in a running reactor it is constantly converted to ^{136}Xe).
- If fuel is fresh (high fission rate) there are insufficient fission fragments for the reactor to be poisoned out.

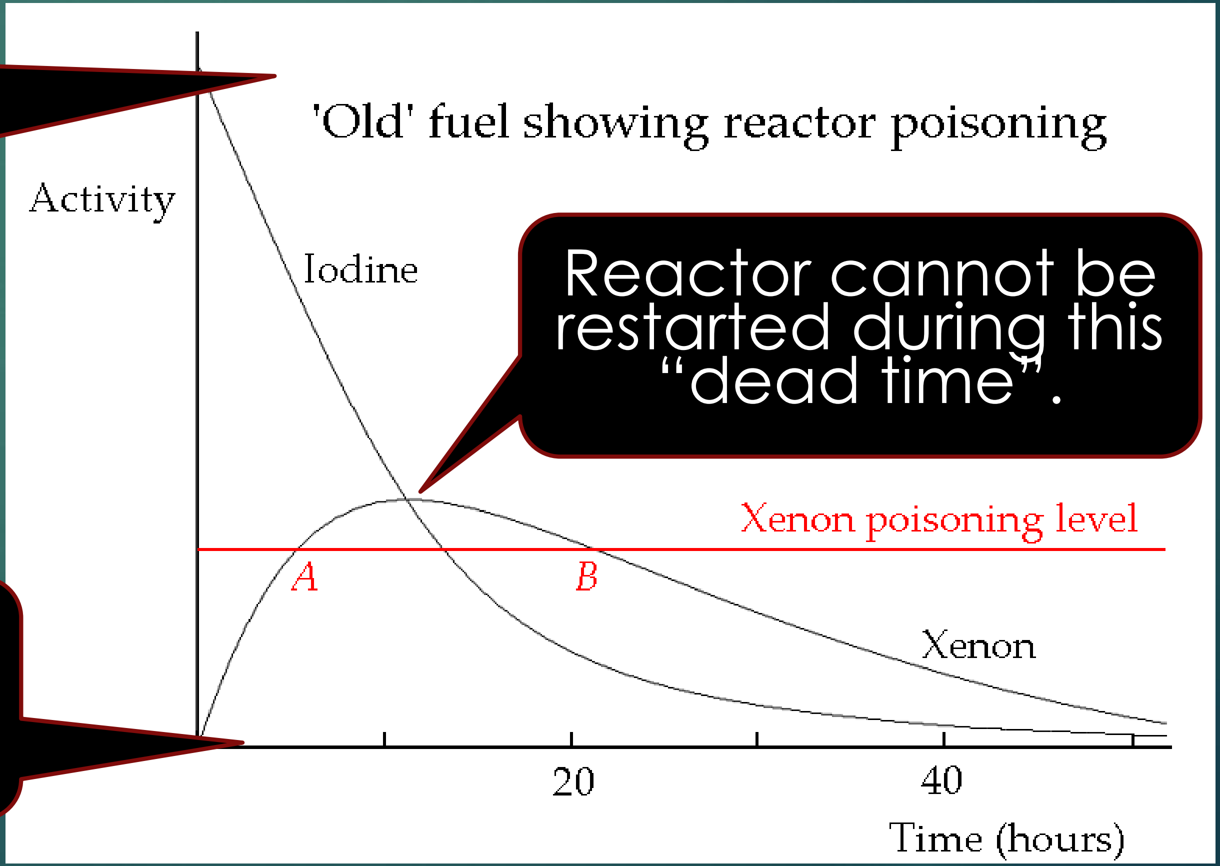


Equilibrium in a running reactor for iodine.

Equilibrium in a running reactor for xenon.

Reactor poisoning

- The equilibrium Xenon accumulation is inversely proportional to the power level (or the neutron flux).
- Hence with old fuel and the same poison level the reactor is poisoned between points *A* and *B*.



Higher equilibrium point in a running reactor for iodine.

Reactor cannot be restarted during this “dead time”.

Equilibrium point in a running reactor for xenon.

Energy deposition in the reactor

- Fission products deposit energy in different ways in a reactor core.
- ***Fission fragments***: heat within fuel elements (most of the fission energy). The elements also distort and swell (damage, also Kr and Xe produced).
- ***Betas and gammas***: heat throughout core - continues from fragments even after shutdown, producing up to 5% of previous operating power.
- ***Neutrons***: heat in moderator. In some early low-temperature reactors using graphite, energy was also stored in the graphite as 'Wigner' energy due to displaced carbon atoms.