MSc Energy (2019/20)

NUCLEAR REACTOR TECHNOLOGY (LECTURE 3)

P. STAMENOV

Time-independent Continuity Equation (Steady-state Reactor Equation)

- In steady state and in 3D:

$$\frac{D}{v}\Delta\Phi(\mathbf{r}) - \Sigma_{\mathrm{a}}(\mathrm{C})\Phi(\mathbf{r}) + S(\mathbf{r}) = 0$$

leackage absorption production

- The macroscopic average absorption in the core: $\Sigma_{\mathrm{a}}(\mathrm{C})$
- In the 'dilute' core limit the diffusion coefficient is essentially the moderator one: $D(\mathrm{M})$
- The infinite neutron multiplication factor is written down as:

$$k_{\infty} \equiv \frac{(\text{total rate of production})}{(\text{total rate of absorption})} = \frac{\int S(\mathbf{r})\mathrm{d}V}{\int \Sigma_{\mathrm{a}}(\mathrm{C})\Phi(\mathbf{r})\mathrm{d}V}$$

Homogenous Reactor Case

- The neutron multiplication rate is essentially independent of position:

$$k_\infty(\mathbf{r}) = \frac{S(\mathbf{r})}{\Sigma_\mathrm{a}(\mathrm{C})\Phi(\mathbf{r})}$$

- In terms of this the reactor equation becomes:

$$\frac{D}{v}\Delta\Phi(\mathbf{r}) - (k_\infty - 1)\Sigma_\mathrm{a}(\mathrm{C})\Phi(\mathbf{r}) = 0$$

- Or explicitly in two terms:

$$\Delta\Phi + \frac{(k_\infty - 1)}{L_\mathrm{c}^2}\Phi = \Delta\Phi + B^2\Phi = 0$$

- With absorption caused by the fuel and moderator only:

$$\Sigma_\mathrm{a}(\mathrm{C}) = \Sigma_\mathrm{a}(\mathrm{F}) + \Sigma_\mathrm{a}(\mathrm{M})$$

$$L_\mathrm{c}^2 = \frac{D}{v\Sigma_\mathrm{a}(\mathrm{C})}$$

$$B^2 = \frac{(k_\infty - 1)}{L_\mathrm{c}^2}$$

Thermal Neutrons Utilisation Factor

- Introducing the thermal neutron utilisation factor *f*:

$$(1-f)=\frac{\Sigma_{\mathrm{a}}(\mathrm{C})-\Sigma_{\mathrm{a}}(\mathrm{F})}{\Sigma_{\mathrm{a}}(\mathrm{C})}$$

- Or the effective ‘homogenous’ diffusion length in the core as a function of the moderator one:

$$L_{\mathrm{c}}^{2}=(1-f)L(\mathrm{M})$$

- Correcting for the ‘slowing-down’ length L_s:

$$\Delta\Phi+B^{2}\Phi=\Delta\Phi+\frac{(k_{\infty}-1)}{(1-f)\left(L^{2}+L_{\mathrm{s}}^{2}\right)}\Phi=0$$

Diffusion and Slowing-Down

Moderator	***D* (cm)**	**L^2 (cm^2)**	**L_s^2 (cm^2)**
H_2O	0.16	8.1	27
D_2O	0.87	30000	131
Graphite	0.84	2650	368

Rectangular or Spherical Symmetry I

- The neutron generation coefficient:

$$\frac{(k_\infty - 1)}{(1-f)\left(L^2 + L_s^2\right)} = B^2$$

- The steady-state reactor equation:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = -B^2 \Phi$$

- Render it separable in three parts: $\Phi = X(x)Y(y)Z(z)$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -\alpha^2 \qquad \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -\beta^2 \qquad \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = -\gamma^2 \qquad B^2 = \alpha^2 + \beta^2 + \gamma^2$$

Rectangular or Spherical Symmetry II

- The solutions are of the form:

$$X(x) = a\sin(\alpha x) + b\cos(\alpha x)$$

- The walls of the reactor must be nodes:

$$b = 0 \quad \alpha L = \pi$$

- The neutron generation coefficient becomes: $$B^2 = 3\frac{\pi^2}{L^2}$$

- The critical volume would be: $$V = L^3 = (3\pi^2)^{3/2} 1/B^3 \approx 161/B^3$$
- In spherical symmetry: $$\frac{\partial^2 u}{\partial r^2} + B^2 u = 0$$
- With solutions of the form:

$$\Phi(r) = u(r)/r = a\sin(Br)/r + b\cos(Br)/r$$

- The flux must be finite at 0 and zero at *R*: $b = 0 \quad BR = \pi$
- The critical volume becomes: $$V = \frac{4}{3}\pi R^3 = \frac{4\pi^4}{3B^3} \approx 130/B^3$$

Sizing a Reactor

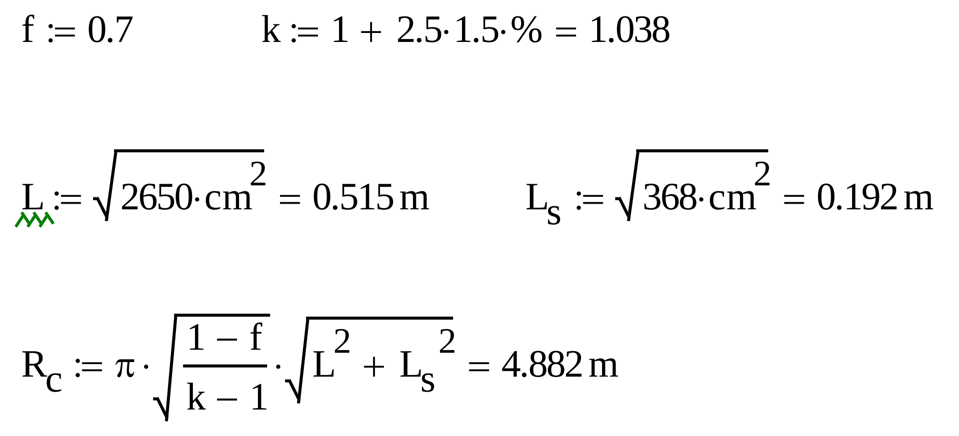
- The neutron generation coefficient is approximately:

$$B^2 = \frac{(k_\infty - 1)}{(1-f)\left(L^2 + L_s^2\right)}$$

- Therefore the critical radius of a spherical reactor would be:

$$R = \frac{\pi}{B} = \pi\sqrt{\frac{1-f}{k_\infty - 1}}\sqrt{L^2 + L_s^2}$$

- For sensible parameters:

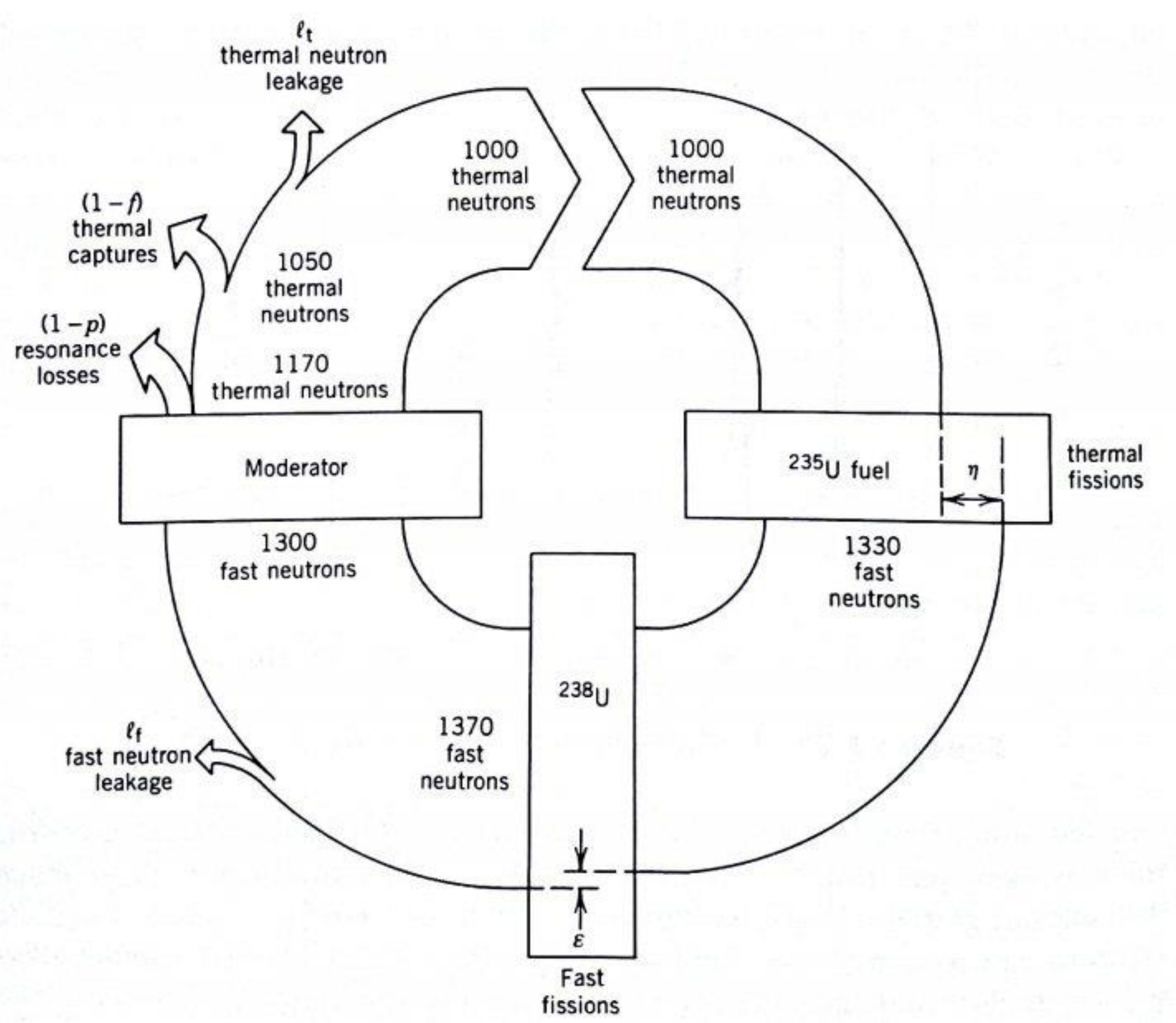


$f := 0.7 \qquad k := 1 + 2.5\cdot 1.5\cdot\% = 1.038$

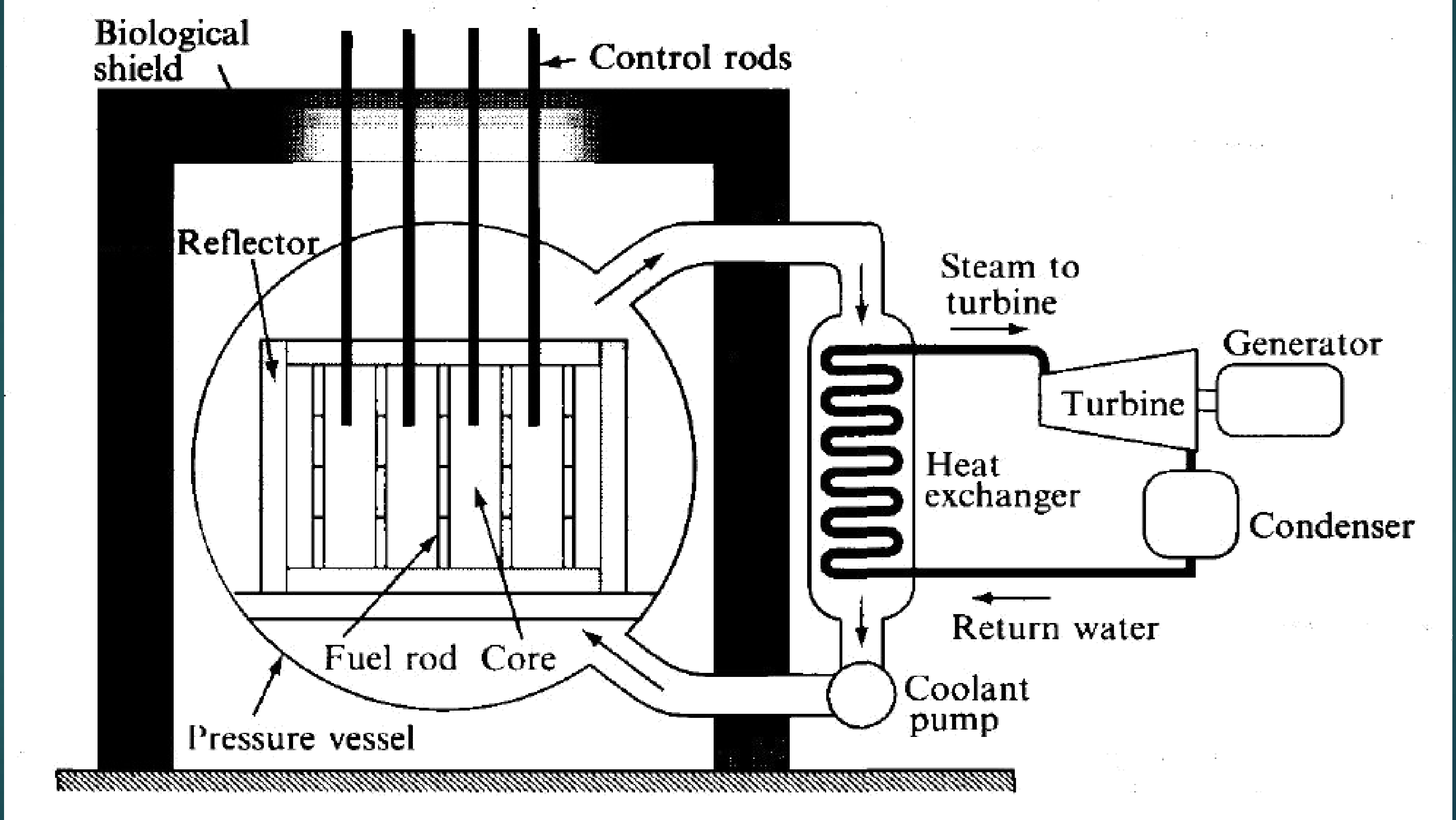
$L := \sqrt{2650\cdot \mathrm{cm}^2} = 0.515\,\mathrm{m} \qquad L_S := \sqrt{368\cdot \mathrm{cm}^2} = 0.192\,\mathrm{m}$

$R_C := \pi\cdot\sqrt{\frac{1-f}{k-1}}\cdot\sqrt{L^2 + L_S{}^2} = 4.882\,\mathrm{m}$

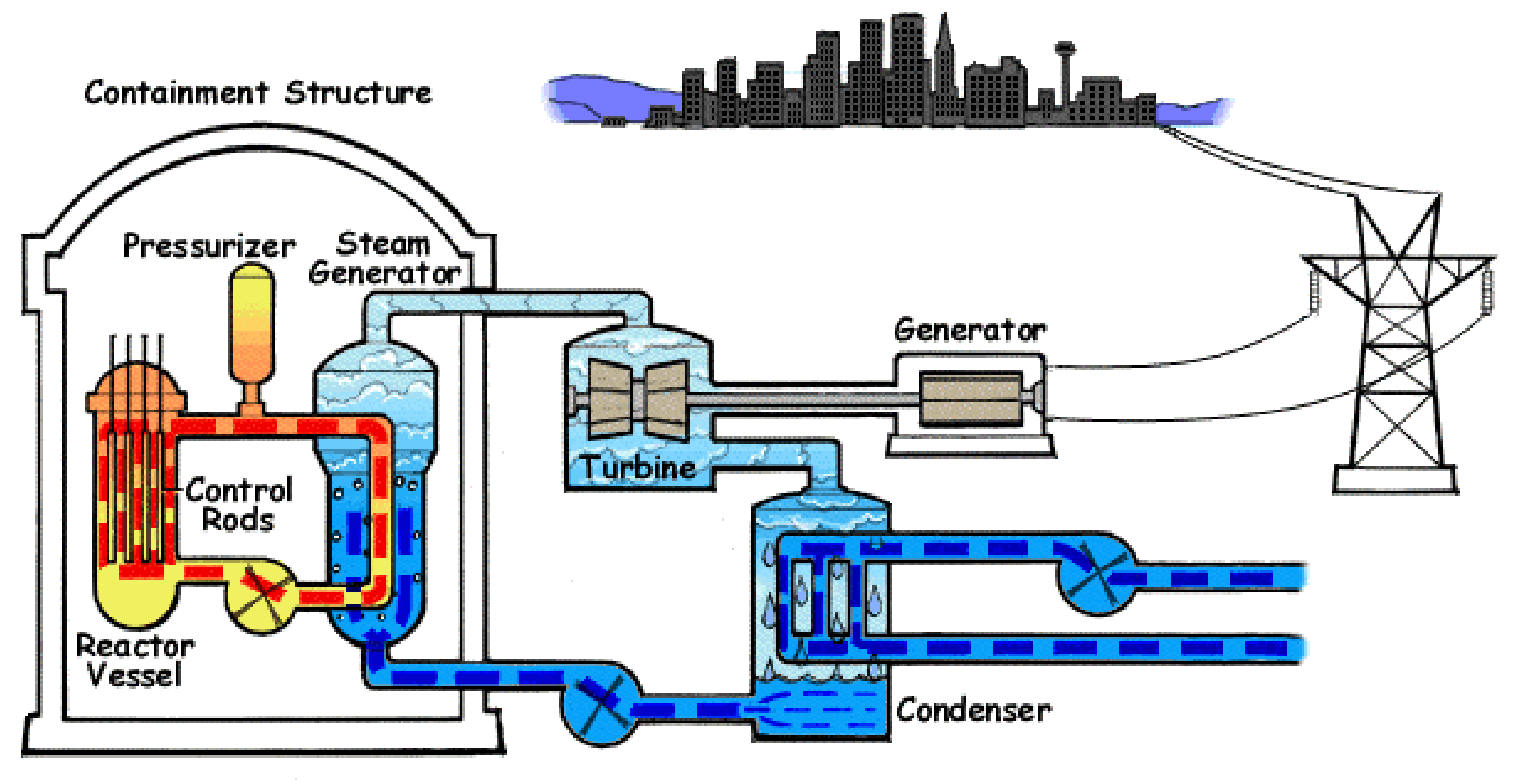
The neutron flux cycle inside a thermal reactor



Schematic of reactor core (PWR)

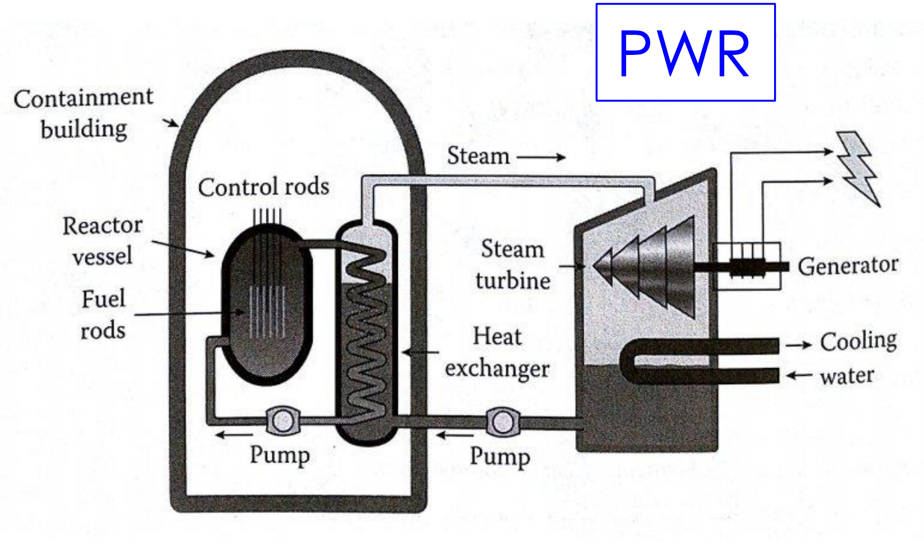


Reactor power plant schematic



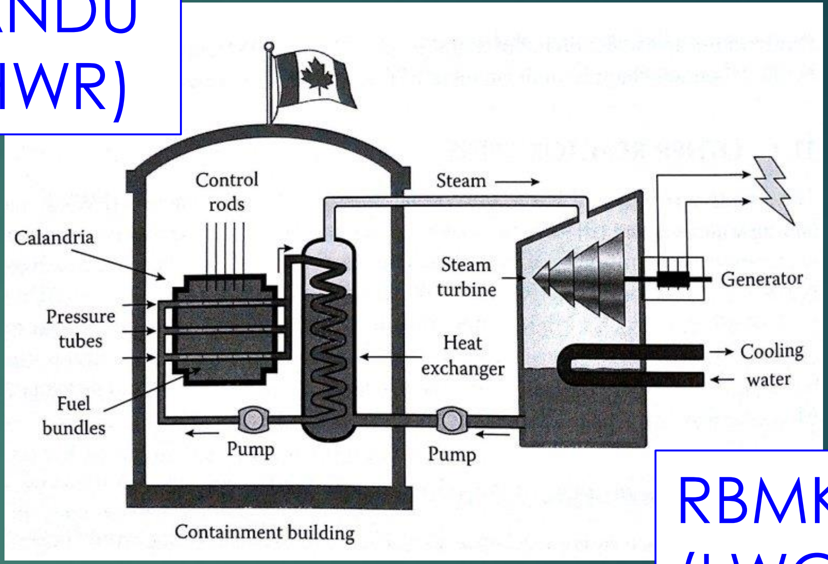
- PWR shown. Reactor vessel includes vertical fuel rods (few cm diameter), control rods and coolant channels, all immersed in water moderator.

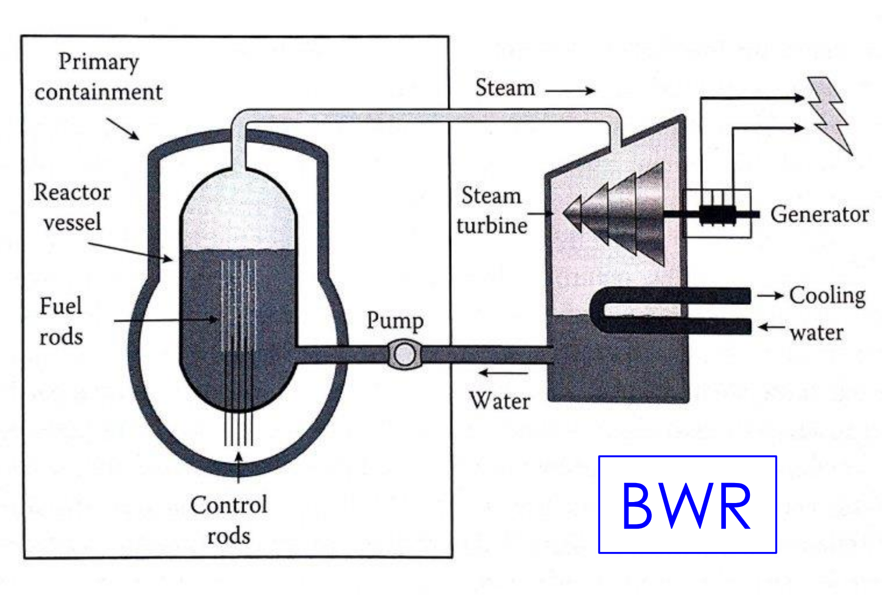
Schematics of power plants based on different reactor types



PWR

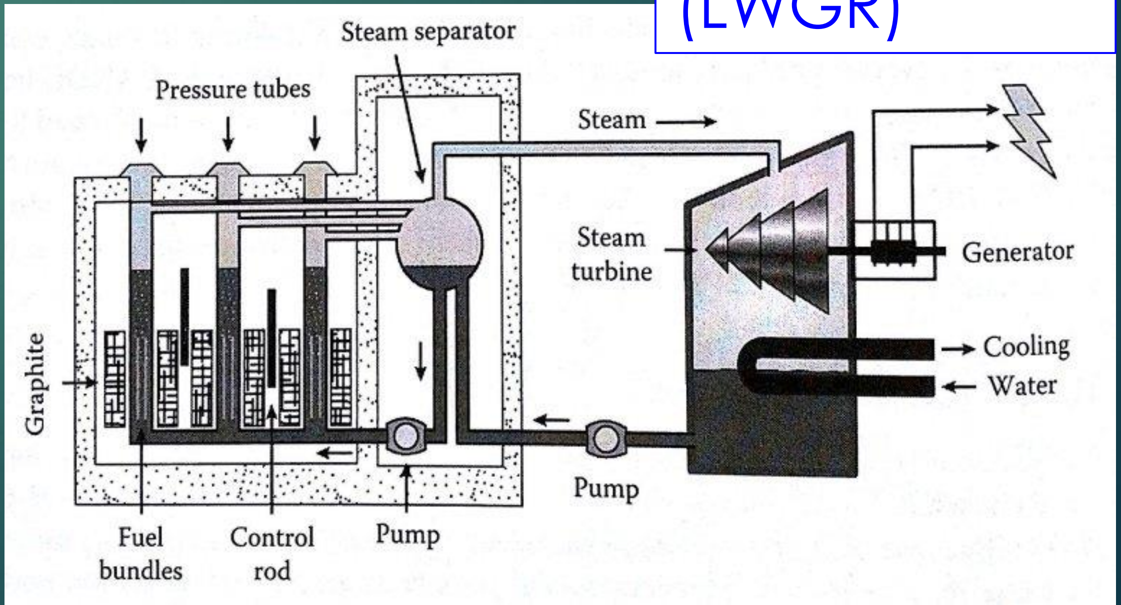
CANDU (PHWR)





BWR

RBMK - РБМК (LWGR)



Newer designs

- New types of power reactors should be safer and more economical, simpler to operate and maintain, and produce less nuclear waste.
- For instance, EPR – European pressurised reactor – a newer type of PWR. The first EPR is being built in at Olkiluoto, Finland and is expected to be operational in 2019.
- The first EPR is already over critical (06/2018) in Taishan, China.

June, 2007

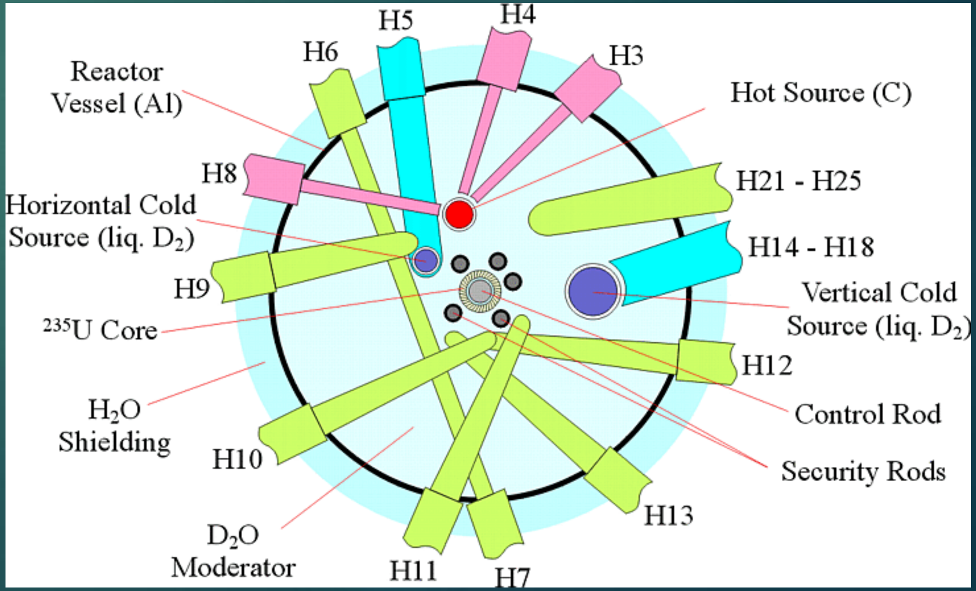
Sept, 2009

Reactors as neutron sources for research

- Some reactors (ILL, Grenoble, France) are specifically designed to produce neutrons for scientific research, often using ***highly enriched fuel*** (~ 90% ^{235}U).
- Intended to produce high neutron densities and fluxes.
- Research includes:
  - the fundamental particle physics of the neutron
  - nuclear fission and fission products
  - neutron diffraction studies of crystals, materials, and chemical and biological molecules…
- See '*Cool things to do with neutrons*', Physics World 26 No 6 (June 2013), p28 – 32.

Institut Laue-Langevin (ILL), Grenoble, France

- International lab (15 countries).
- Adjacent to the ESRF synchrotron →
- Single fuel element (highly enriched uranium > 97% ^{235}U) + heavy water moderator and coolant.
- Very high neutron fluxes – 1.5×10^{15} neutrons per second per cm^2
- 50 experimental set-ups
- Thermal power 58.3 MW



Inside the ILL

- Inside the containment building for the ILL reactor, showing the fission fragment spectro-meter *Lohengrin*.

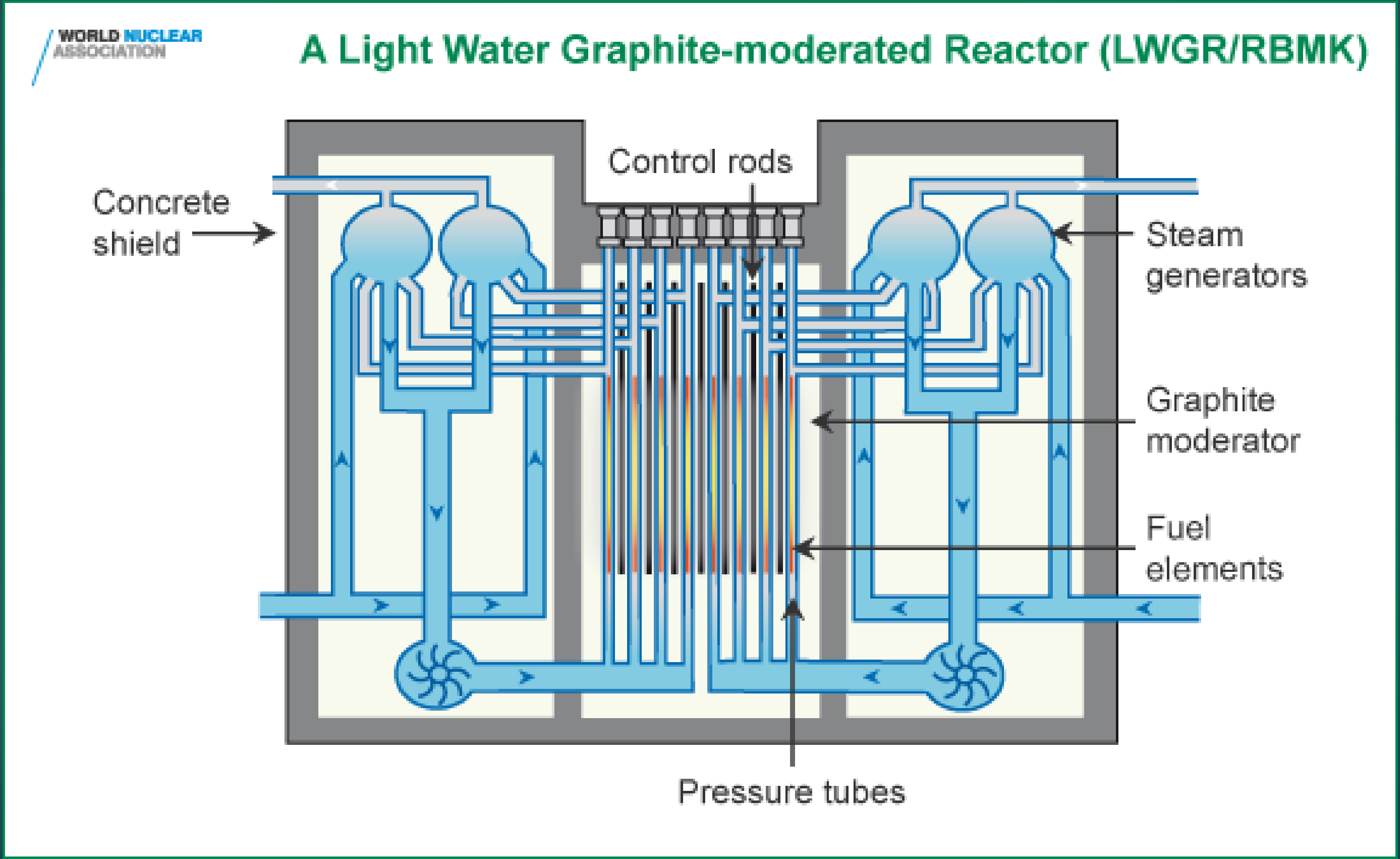
Inside the ILL

- Looking down into the reactor pool, showing blue Čerenkov radiation from the high energy β-rays.

- Storage pool next to reactor, containing used fuel elements (still glowing in blue).

The Chernobyl reactor (RMBK)

- Huge! Uses over 100 tonnes of U. 1.5GW electricity.
- Design flaws, positive void coefficient



Plutonium production

- Produced to some extent in all power reactors:

$$^{238}_{92}\mathrm{U} + \mathrm{n} \rightarrow {}^{239}_{92}\mathrm{U} + \gamma$$

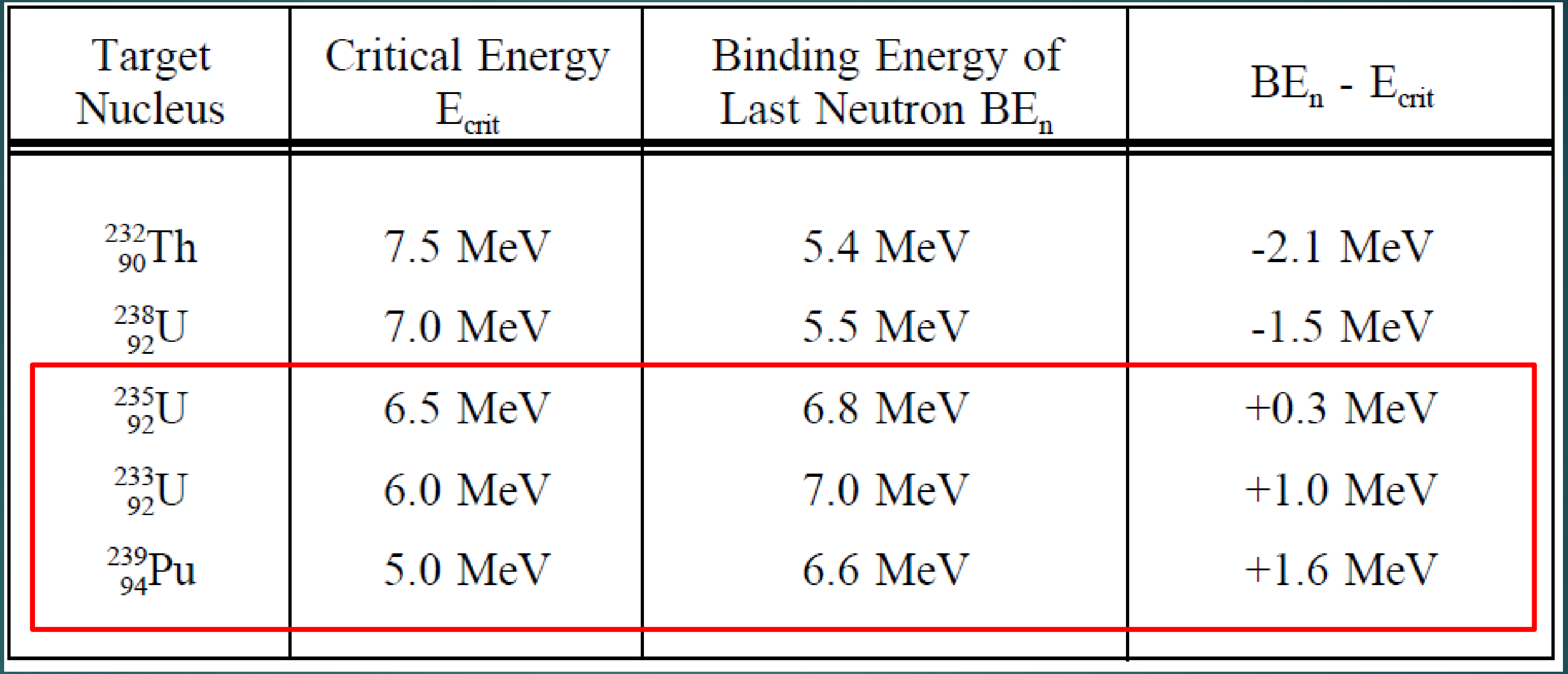
$$^{239}_{92}\mathrm{U} \rightarrow {}^{239}_{93}\mathrm{Np} + \mathrm{e}^- + \bar{\nu} \qquad t_{1/2} = 23\ \mathrm{min}$$

$$^{239}_{93}\mathrm{Np} \rightarrow {}^{239}_{94}\mathrm{Pu} + \mathrm{e}^- + \bar{\nu} \qquad t_{1/2} = 2.4\ \mathrm{days}$$

Typically ~5g/kgU in LWR

- $t_{1/2}$ (^{239}Pu) = 24,000 years (alpha decay)
- ^{239}Pu is even-odd. Like ^{235}U, its cross section for fission by slow neutrons is large, they are both ***fissile***).
- ^{239}Pu can be mixed into fuel (‘MOX’) in reactors using thermal neutrons (original ^{238}U is even-even and is not fissile).

Comparison of critical energies



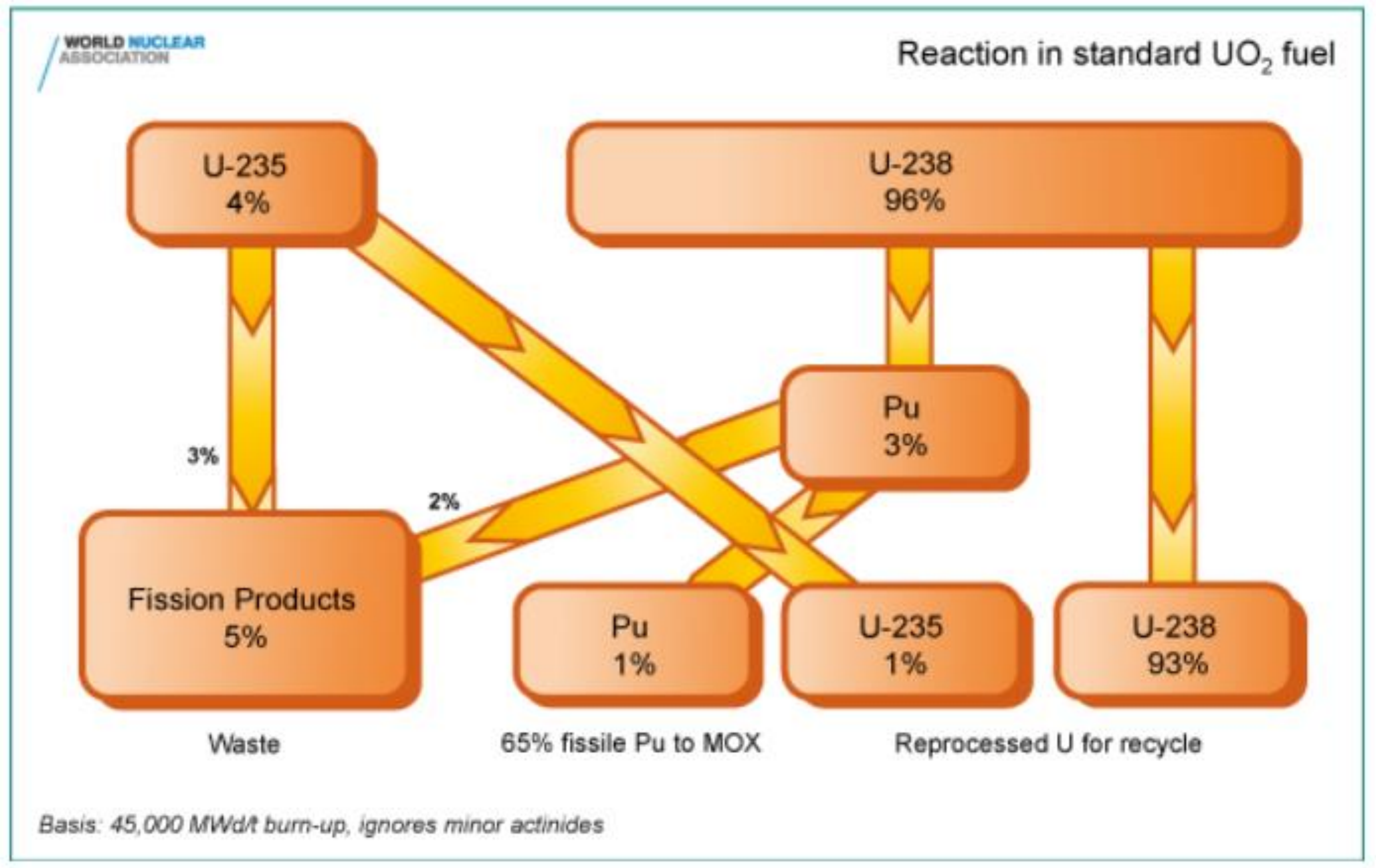
Target Nucleus	Critical Energy E_{crit}	Binding Energy of Last Neutron BE_n	$BE_n - E_{crit}$
$^{232}_{90}\text{Th}$	7.5 MeV	5.4 MeV	-2.1 MeV
$^{238}_{92}\text{U}$	7.0 MeV	5.5 MeV	-1.5 MeV
$^{235}_{92}\text{U}$	6.5 MeV	6.8 MeV	+0.3 MeV
$^{233}_{92}\text{U}$	6.0 MeV	7.0 MeV	+1.0 MeV
$^{239}_{94}\text{Pu}$	5.0 MeV	6.6 MeV	+1.6 MeV

- The bottom three do not need the kinetic energy of the neutrons to undergo fission. They are fissile.

Plutonium – ^{239}Pu

- Potentially a very large source of energy, as natural uranium is > 99% ^{238}U (it does contribute to reactor's power output).
- Large stockpiles worldwide of uranium depleted in ^{235}U, produced by enrichment programmes.
- Amount of ^{239}Pu produced depends on reactor type.
- For a given amount of ^{235}U (power), 4 times more ^{239}Pu produced from natural fuel compared to enriched.
  - (natural U reactors: Magnox, CANDU and Chernobyl-type)
- Much larger amounts of ^{239}Pu can be produced (over several years), greater than the ^{235}U amount consumed, in '*breeder reactors*'.

Plutonium production in LWR



Basis: 45,000 MWd/t burn-up, ignores minor actinides

Breeder reactors

- Generate more fissile material than is consumed.
- Use fast neutrons for the chain reaction, despite the small fission cross-sections.
- Very fuel-efficient (utilise nearly 100% of the energy stored in the fuel).

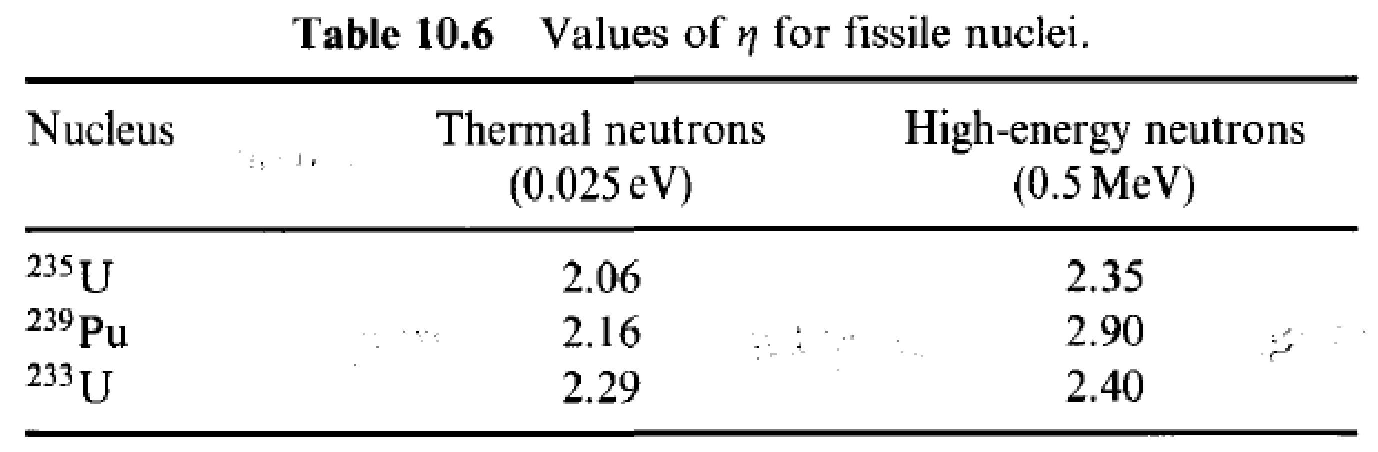


Table 10.6 Values of η for fissile nuclei.

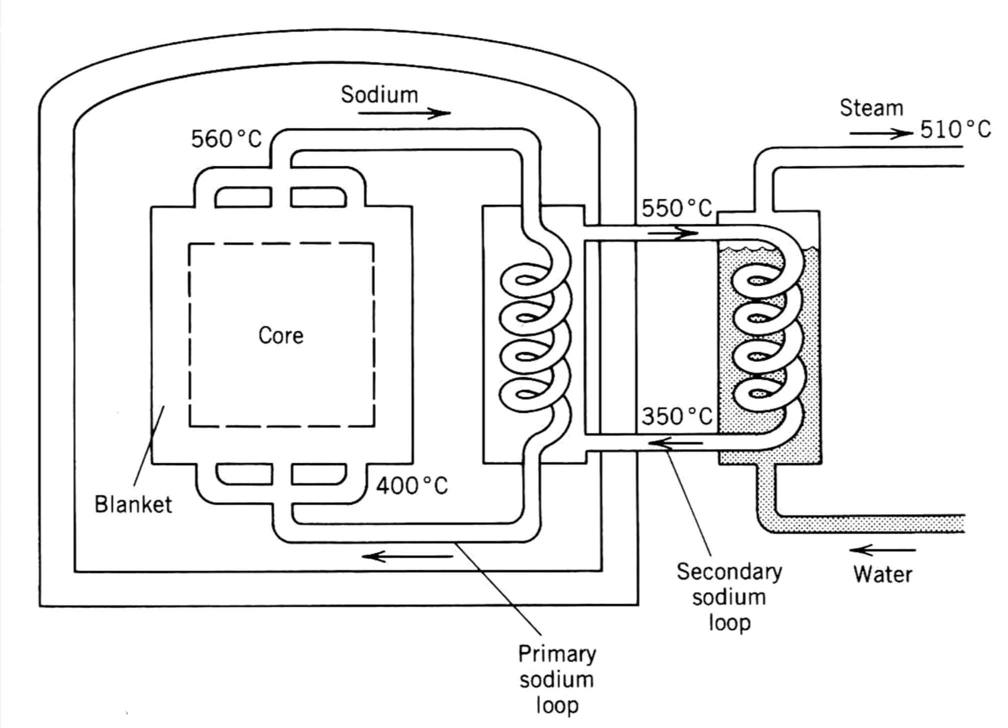
Nucleus	Thermal neutrons (0.025 eV)	High-energy neutrons (0.5 MeV)
^{235}U	2.06	2.35
^{239}Pu	2.16	2.90
^{233}U	2.29	2.40

Yield: η = (number of new neutrons)/(absorbed neutrons)

- Sustainable breeding possible if $\eta > \underbrace{1}_{\text{for fission}} + \underbrace{1}_{\text{for breeding}} + \underbrace{0.2}_{\text{losses}}$

Breeder reactor structure

- Small core with fuel enriched to around 20% ^{235}U.
- No moderator
- Very high power density in the core
- Liquid metal coolant for efficient heat removal & low absorption.



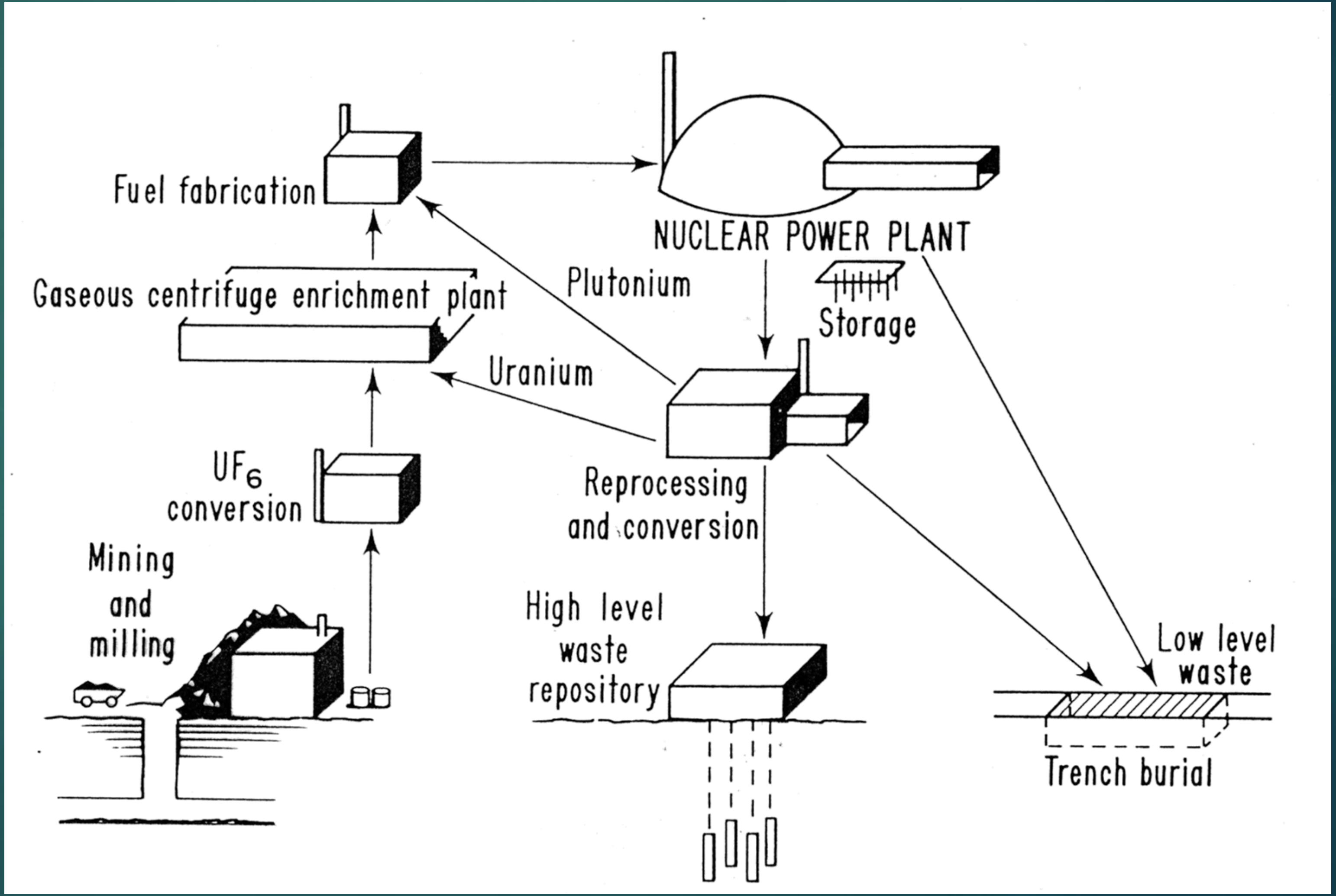
- Natural or depleted uranium ‘blanket’ around core, in which the ^{239}Pu is bred.
- No pressurised reactor vessel needed.

More about plutonium

- Breeder reactor technology is difficult, e.g. a liquid sodium coolant is required (because the core energy density produced is so high).
- ^{239}Pu is extremely radiotoxic, half-life = 24,100 y.
- More uranium deposits have now been discovered, so the need for ^{239}Pu is less.
- More ^{239}Pu available because of weapons decommissioning – so is breeding needed?
- ^{239}Pu is a weapons material (e.g. Nagasaki), like ^{235}U (Hiroshima), but unlike ^{238}U.
- Unlike ^{235}U, ^{239}Pu needs merely chemical separation to isolate it.

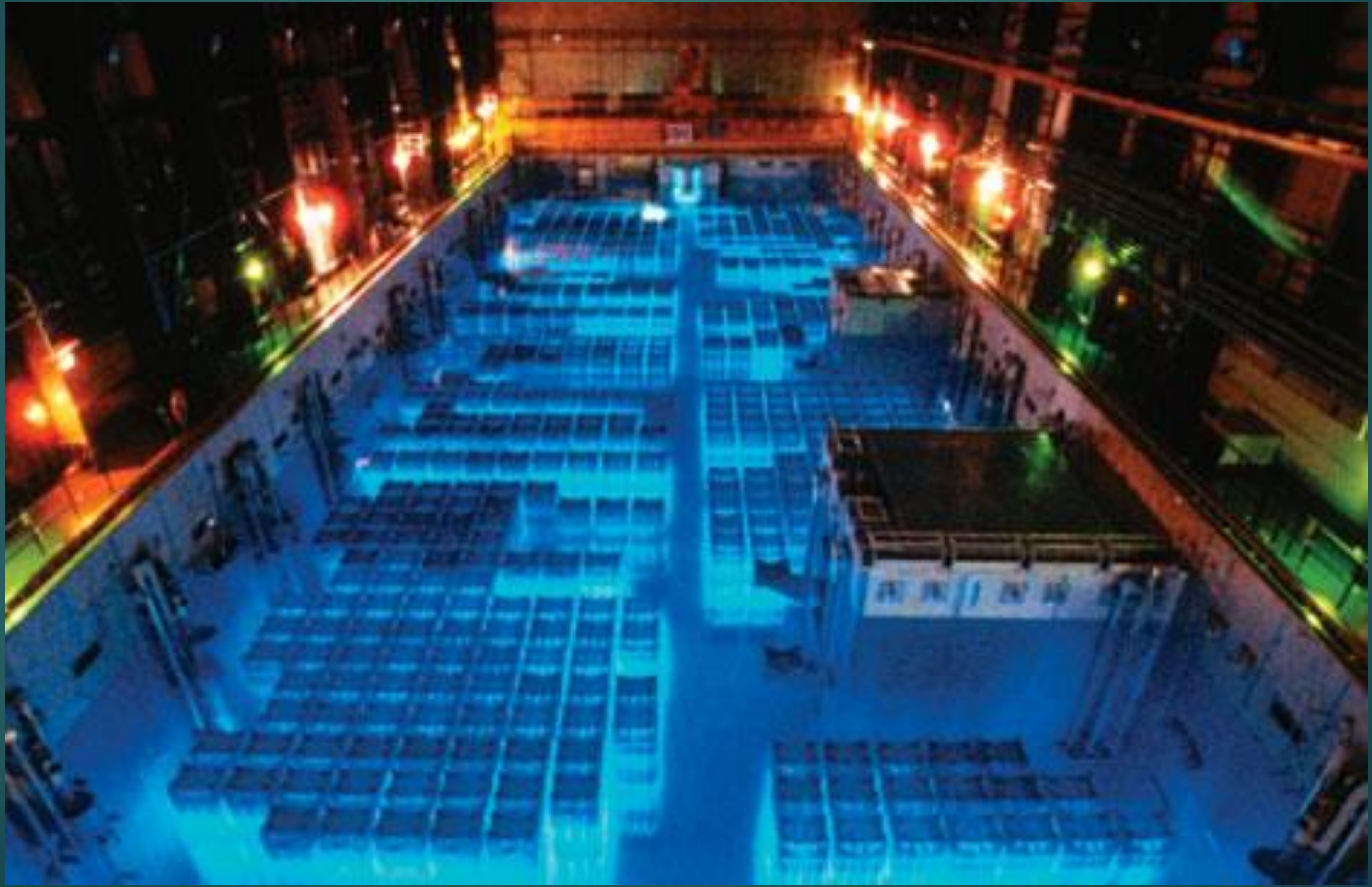
The nuclear fuel cycle

- Reactors depend on facilities like ore mining, milling, ^{235}U enrichment, fuel rod fabrication, used fuel reprocessing and waste disposal.



Spent fuel cooling

- Cooling pool in Cap de la Hague, Cherbourg, France

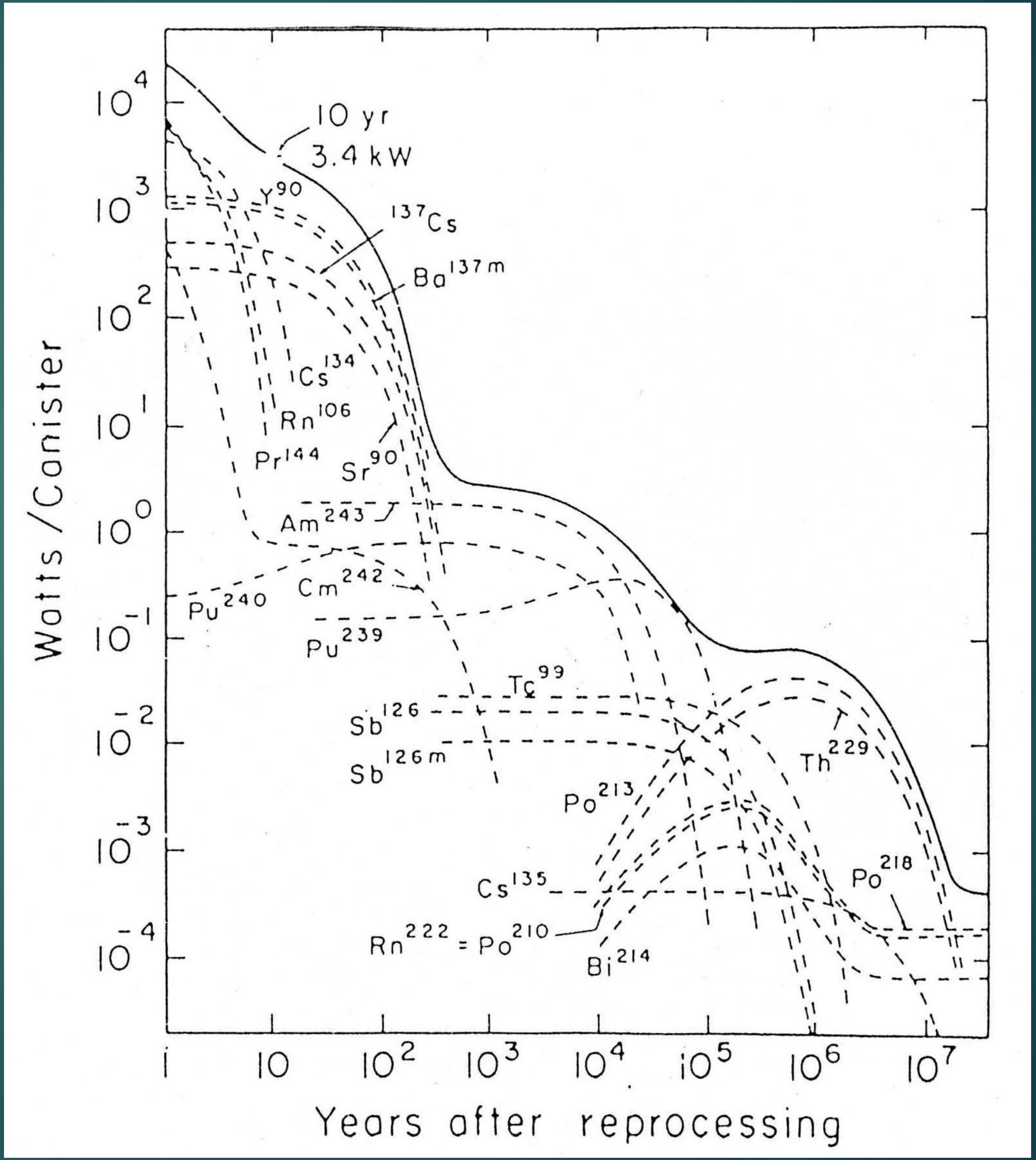


Waste storage

- Canadian waste repository; the concrete containers have walls 1 m thick and are designed to last for 100 years.

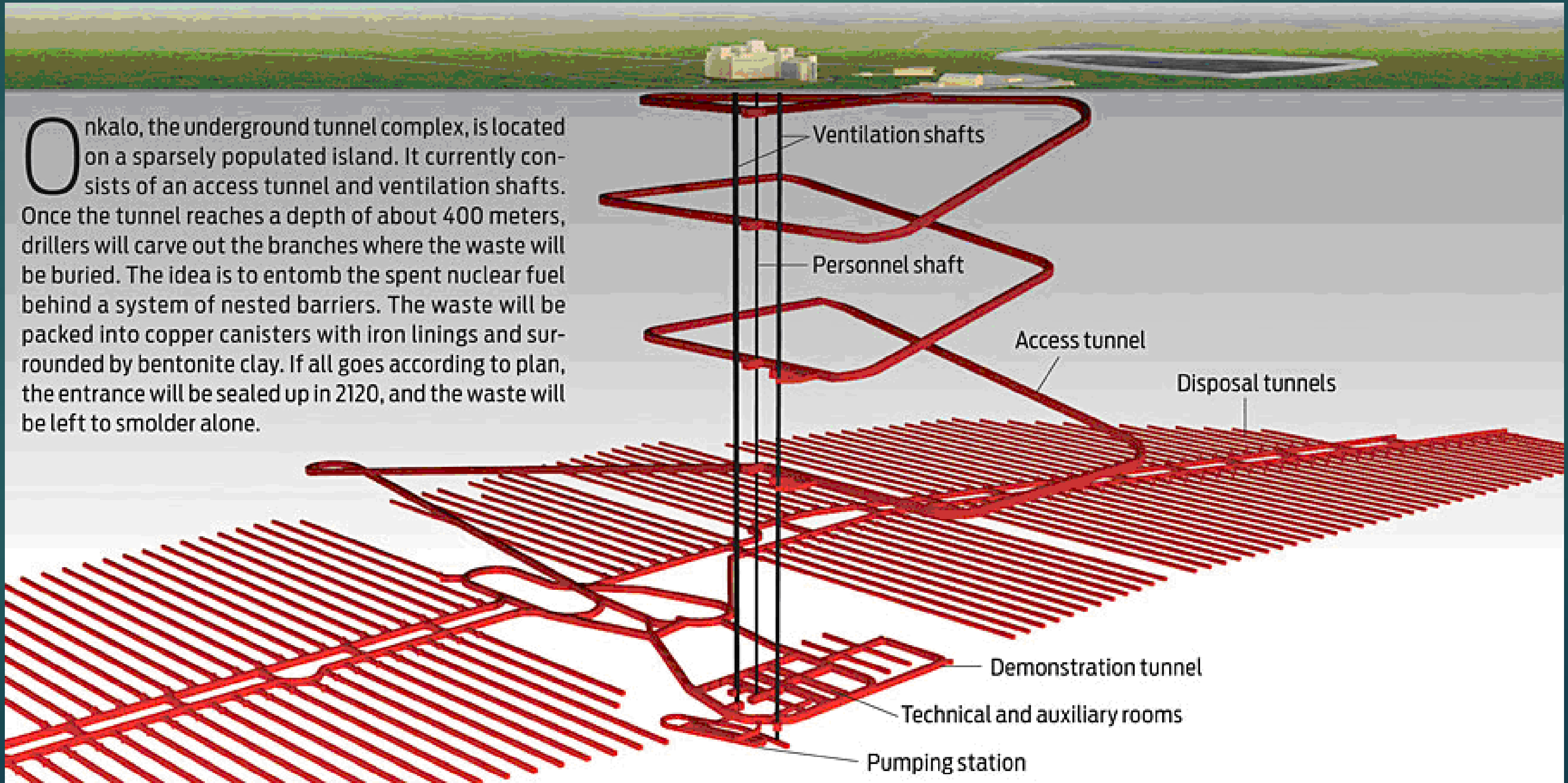
Fuel waste activity

- Power vs time from fission fragment and actinide decays produced by one month's waste from a 1 GW reactor.
- After a few hundred years most of the fission fragments will have decayed, leaving behind actinide activities comparable with that of the original unirradiated uranium fuel.



Long term fuel disposal

- It would appear that, ultimately, high-level waste ought to be buried deep underground in (vitrified) glassified form.
- Finland is planning a deep repository near the site of the Olkiluoto reactors themselves.



Onkalo, the underground tunnel complex, is located on a sparsely populated island. It currently consists of an access tunnel and ventilation shafts. Once the tunnel reaches a depth of about 400 meters, drillers will carve out the branches where the waste will be buried. The idea is to entomb the spent nuclear fuel behind a system of nested barriers. The waste will be packed into copper canisters with iron linings and surrounded by bentonite clay. If all goes according to plan, the entrance will be sealed up in 2120, and the waste will be left to smolder alone.