MSc Energy (2019/20)

NUCLEAR REACTOR TECHNOLOGY (LECTURE 2)

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Neutron Diffusion

- 1D Diffusion Equation $$J_z = -D\frac{\partial n}{\partial z}$$
- 1st Fick's Law
- 3D Diffusion Equation $\mathbf{J} = \mathbf{i}J_x + \mathbf{j}J_y + \mathbf{k}J_z = -\hat{D}\vec{\nabla} n$
- Relation of the diffusion coefficient to the scattering length

$$J_z = \frac{v}{6}n(z-\lambda_s) - \frac{v}{6}n(z+\lambda_s) = -\frac{v\lambda_s}{3}\frac{\partial n}{\partial z} = -D\frac{\partial n}{\partial z}$$

$$D = \frac{v\lambda_s}{3}$$

Continuity Equation

- Second Fick's Law in 3D: $$\frac{\partial n}{\partial t} = -\left[\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}\right] = -\vec{\nabla}\bullet\mathbf{J}$$

- The same in 1D (attention on the *D* not being constant):

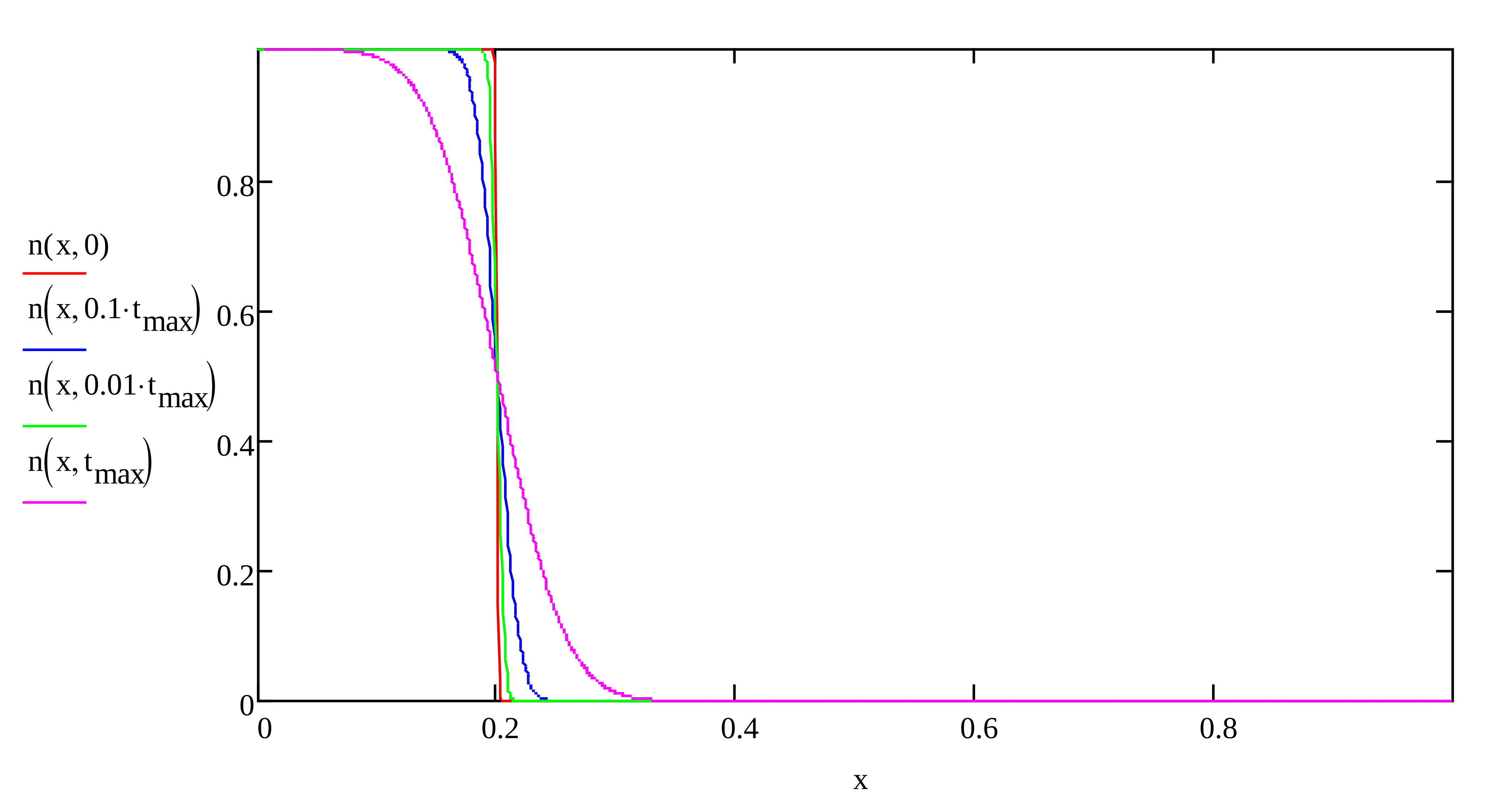
$$\frac{\partial n}{\partial t} = D\nabla^2 n = \Delta D n$$

- Time-dependent continuity equation for the neutron flux: $$\Phi(\mathbf{r}) = v n(\mathbf{r})$$

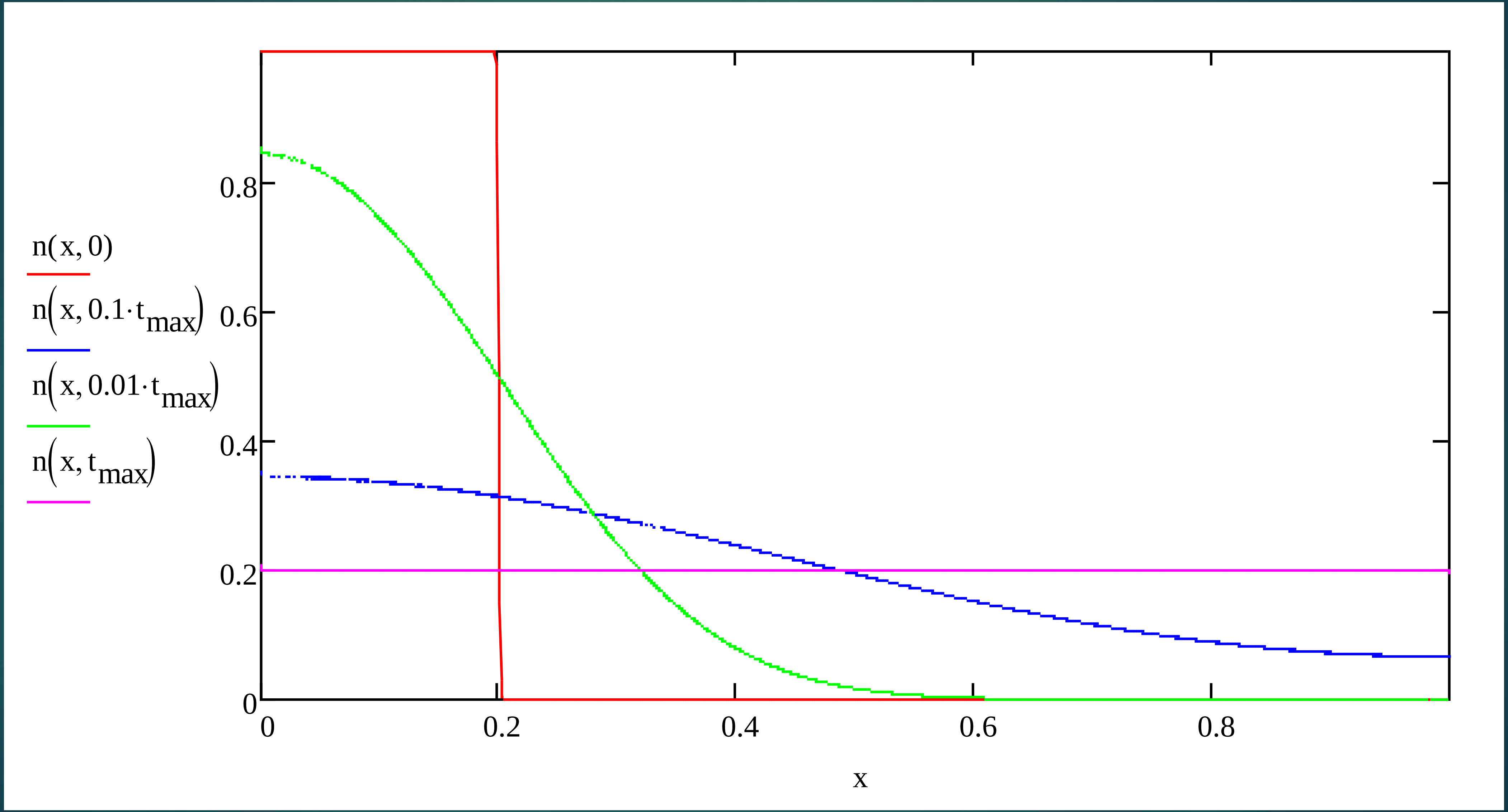
$$\frac{\partial n(\mathbf{r},t)}{\partial t} = D\nabla^2 n(\mathbf{r},t) - \Sigma_a \Phi(\mathbf{r},t) + S(\mathbf{r},t)$$

- Total macroscopic absorption cross section for nuclear density *N*: $$\Sigma_a = N\sigma_a$$

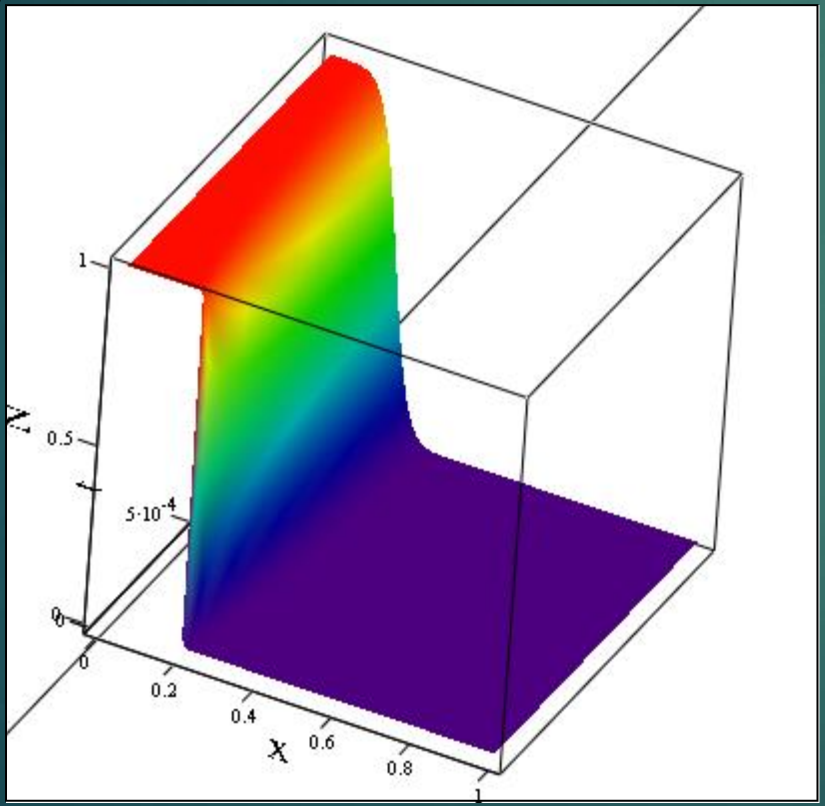
Constant or Infinite Source

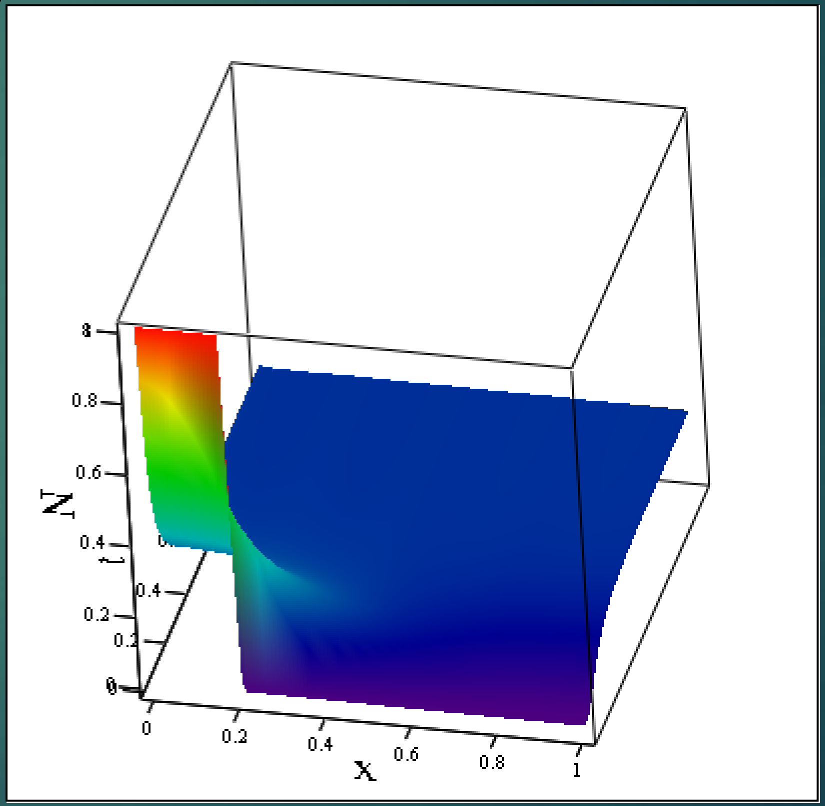


Finite Source

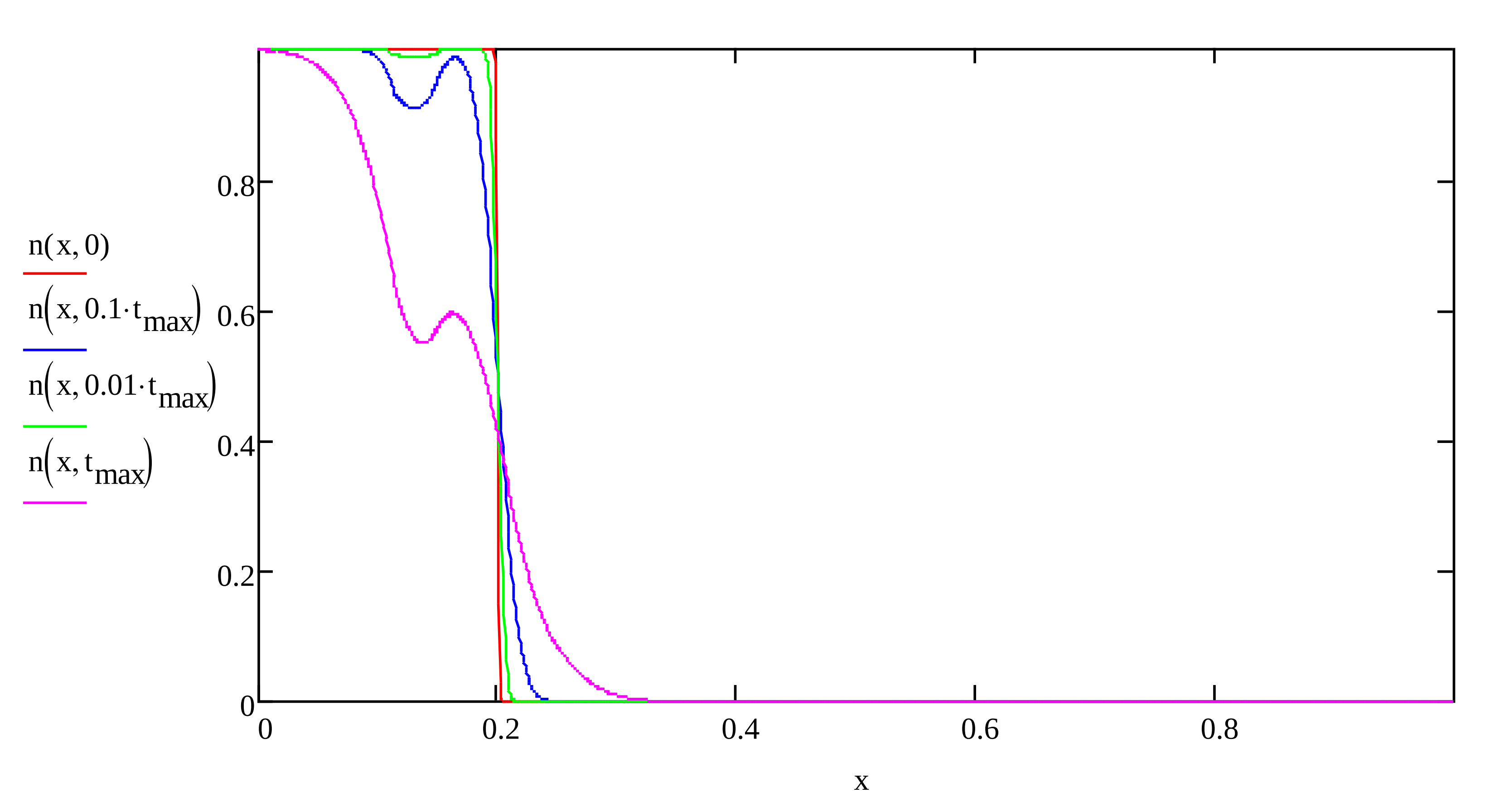


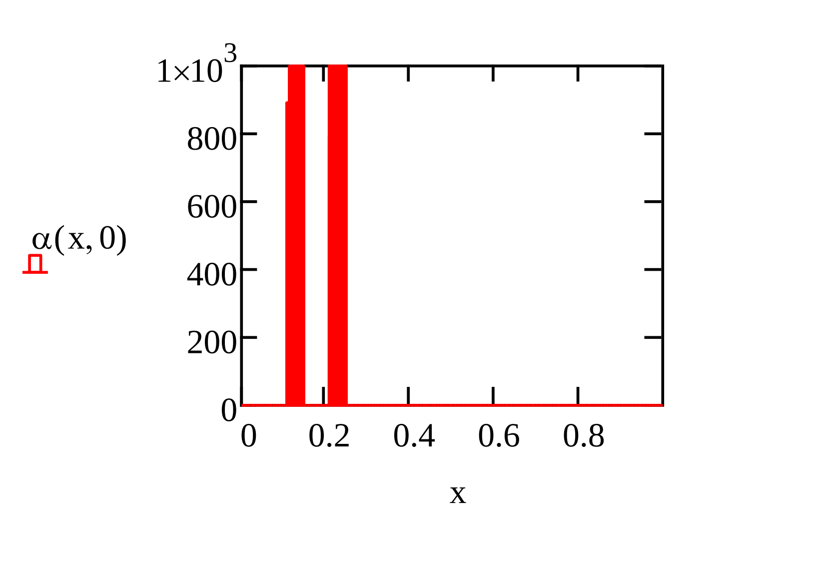
Constant and Finite Source Solutions (only diffusion) – Side by Side



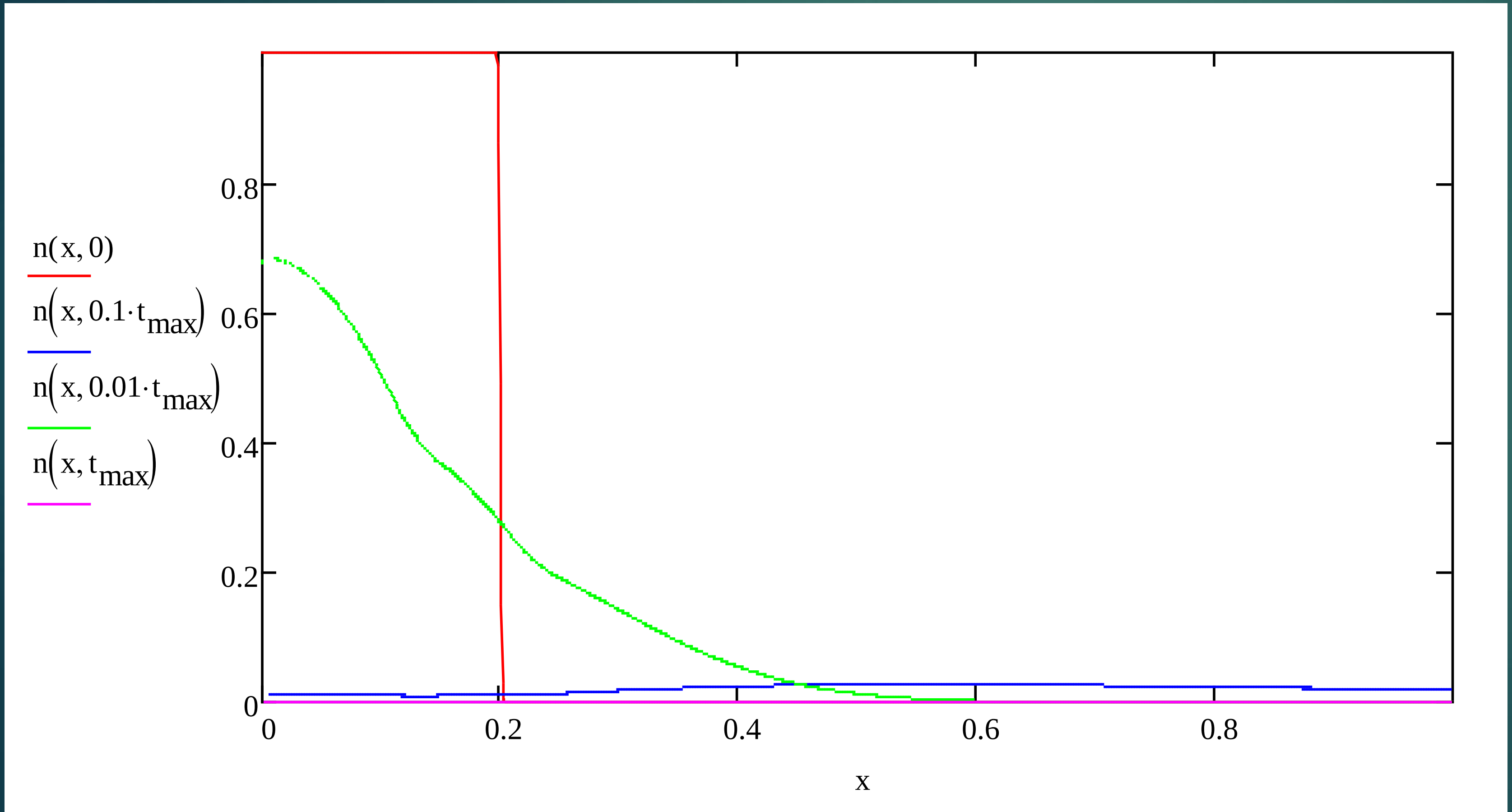


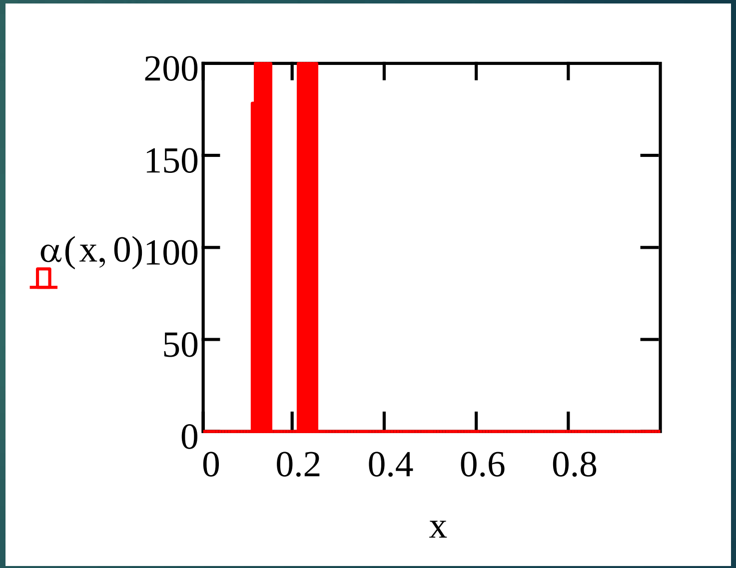
Continuity Equation with Absorption: Constant Source Case



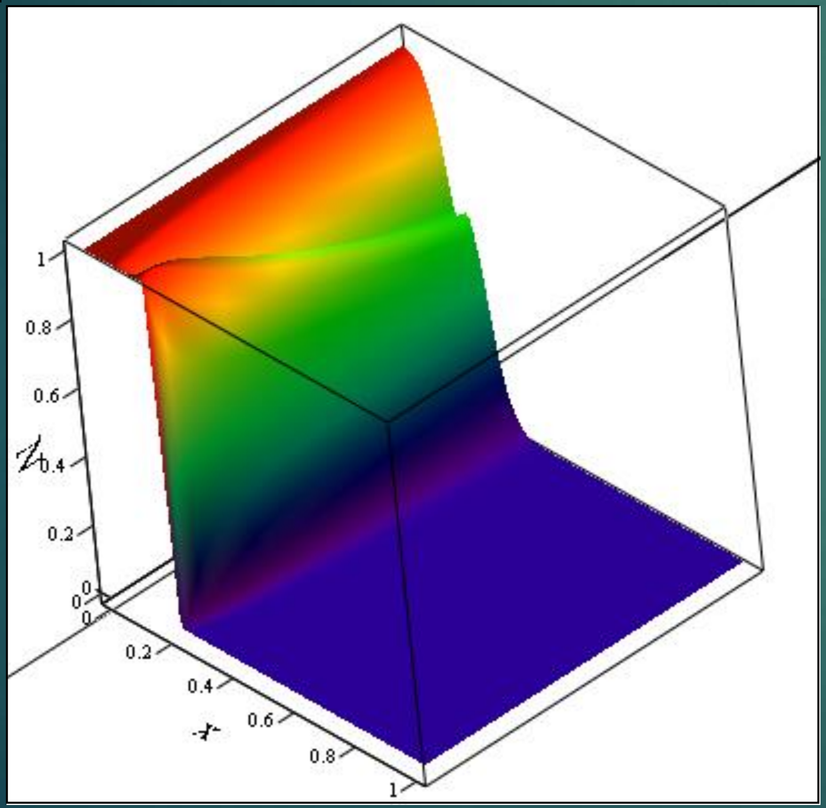


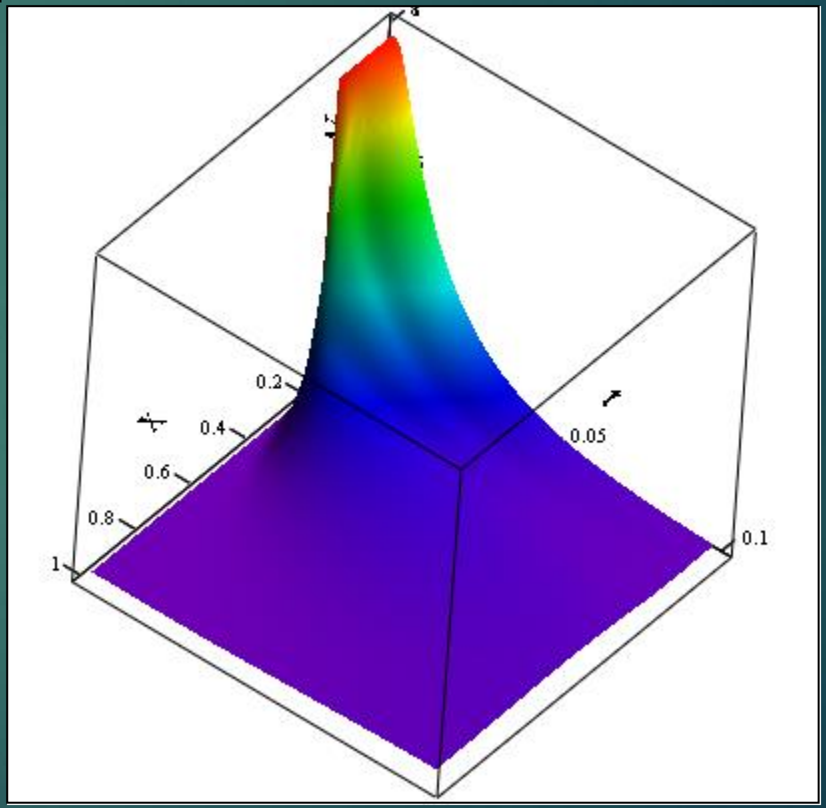
Continuity Equation with Absorption: Finite Source Case





Constant and Finite Source Solutions (with absorption) – Side by Side





Steady-state Continuity Equation

- With no sources, other than at the boundary **r** = 0, within the moderator M:

$$\Delta\Phi(\mathbf{r}) = \frac{v\Sigma_a(\mathrm{M})}{D}\Phi(\mathbf{r}) = \frac{\Phi(\mathbf{r})}{L^2}$$

- Where *L* is nothing but the diffusion length in the moderator M:

$$L^2 = \frac{D}{v\Sigma_a(\mathrm{M})}$$

- Another relation can be established with the neutron ‘lifetime’ in the moderator *τ*:

$$\sqrt{\frac{D}{v\Sigma_a(\mathrm{M})}} = \sqrt{D\tau}$$

$$\tau = \frac{1}{v\Sigma_a(\mathrm{M})}$$

The Gradient Operator in Different Coordinate Systems

- Decartian:

$$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$$

- Cylindrical:

$$\nabla f = \frac{\partial f}{\partial \rho}\mathbf{e}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\mathbf{e}_\varphi + \frac{\partial f}{\partial z}\mathbf{e}_z$$

- Spherical:

$$\nabla f = \frac{\partial f}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\mathbf{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\mathbf{e}_\varphi$$

The Laplace Operator in Different Coordinate Systems

- Decartian:

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

- Cylindrical:

$$\Delta f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

- Spherical:

$$\Delta f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\left(\sin\theta \frac{\partial^2 f}{\partial \varphi^2}\right)$$

Stationery Isotropic Diffusion in Spherical Coordinates

- The Laplacian equation becomes:

$$\Delta\Phi = \frac{\partial^2\Phi}{\partial r^2} + \frac{2}{r}\frac{\partial\Phi}{\partial r} = \frac{\Phi}{L^2}$$

- Substituting in with a transformed flux:

$$u(r) = r\Phi(r)$$

$$\frac{\partial^2 u(r)}{\partial r^2} + \frac{u(r)}{L^2} = 0$$

- The general solution is:

$$u(r) = ae^{-r/L} + be^{+r/L}$$

$$\Phi(r) = \frac{ae^{-r/L}}{r} + \frac{be^{+r/L}}{r}$$

$$\lim_{r\to\infty}\Phi(r) = 0 \Rightarrow b = 0$$

Mean Square Distance Travelled by a Neutron in Medium

- Performing the averaging:

$$\langle r^2 \rangle = \frac{\int_0^\infty r^2 \Phi(r) \mathrm{d}V}{\int_0^\infty \Phi(r) \mathrm{d}V} = \frac{\int_0^\infty r^3 e^{-r/L} \mathrm{d}r}{\int_0^\infty r e^{-r/L} \mathrm{d}r} = 6L^2$$

- With a volume element:

$$\mathrm{d}V = 4\pi r^2 \mathrm{d}r$$

- The root mean square distance is therefore:

$$r_{\mathrm{rms}} = \sqrt{6}L \cong 2.5L$$