PY2N20
Material Properties and Phase Diagrams
Lecture 4

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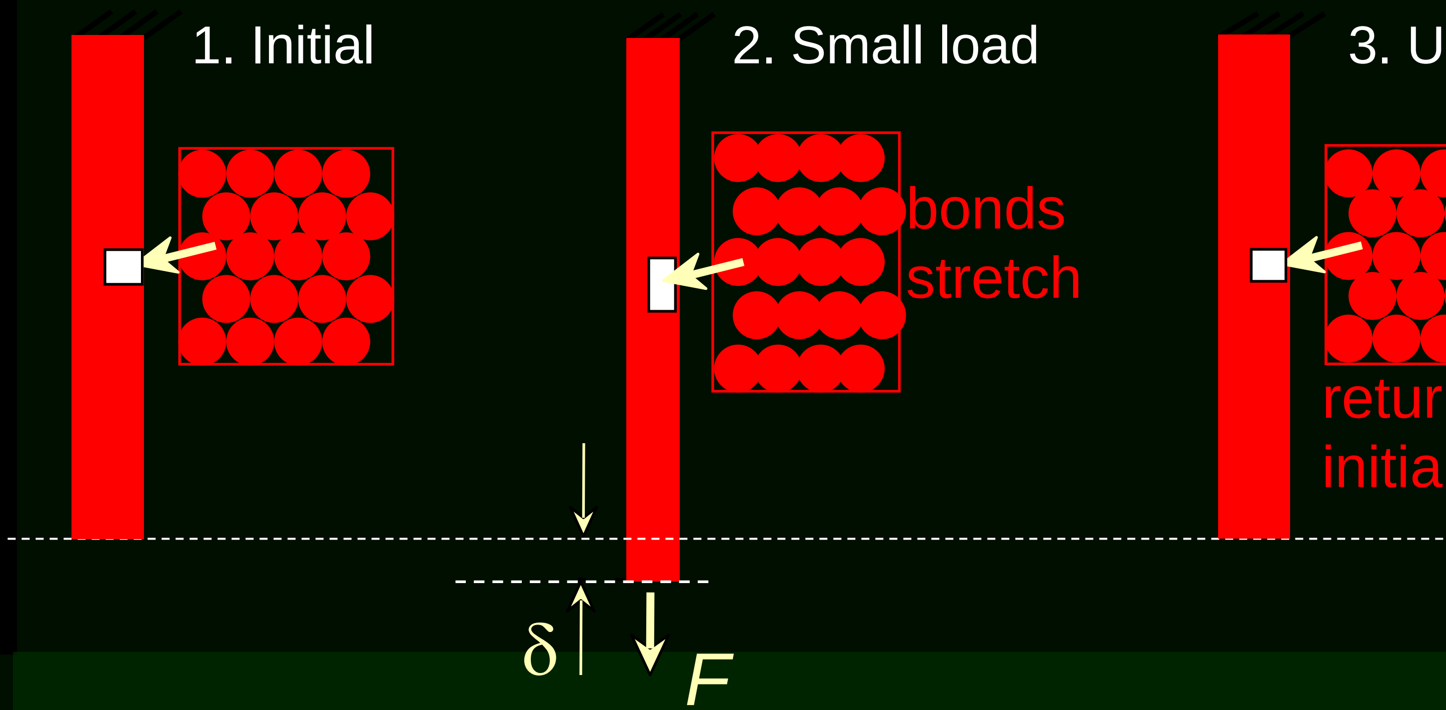
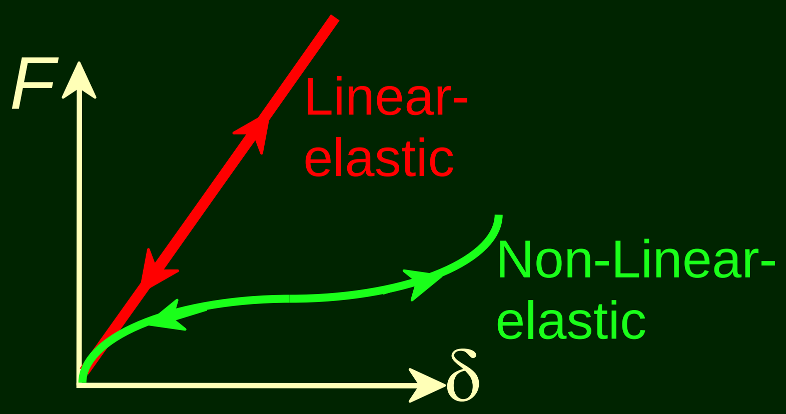
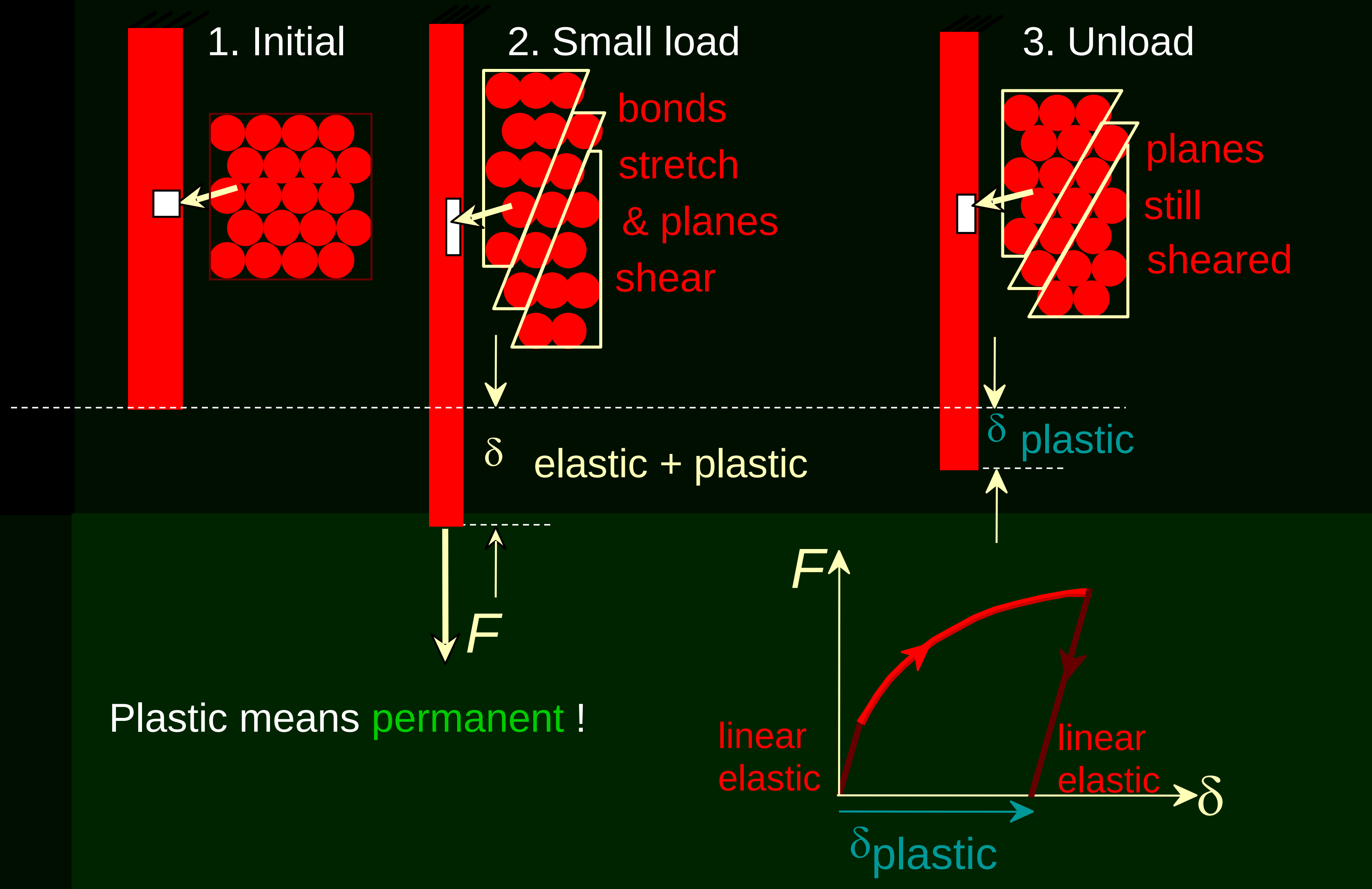
Elastic Deformation



Elastic means reversible!



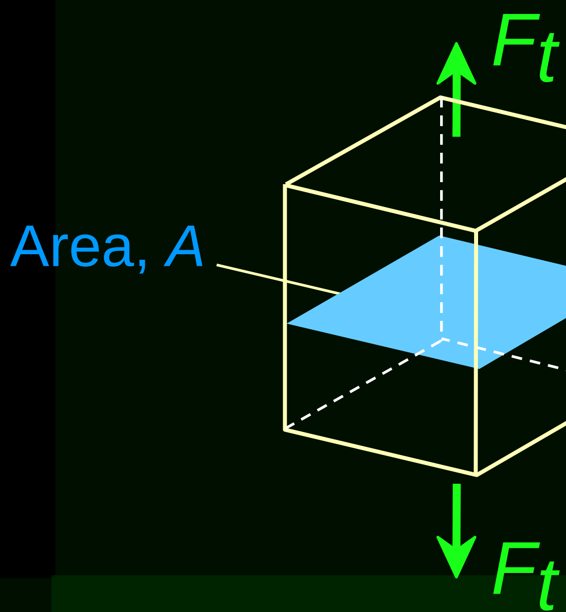
Plastic Deformation (Metals)



Plastic means permanent !

Engineering Stress

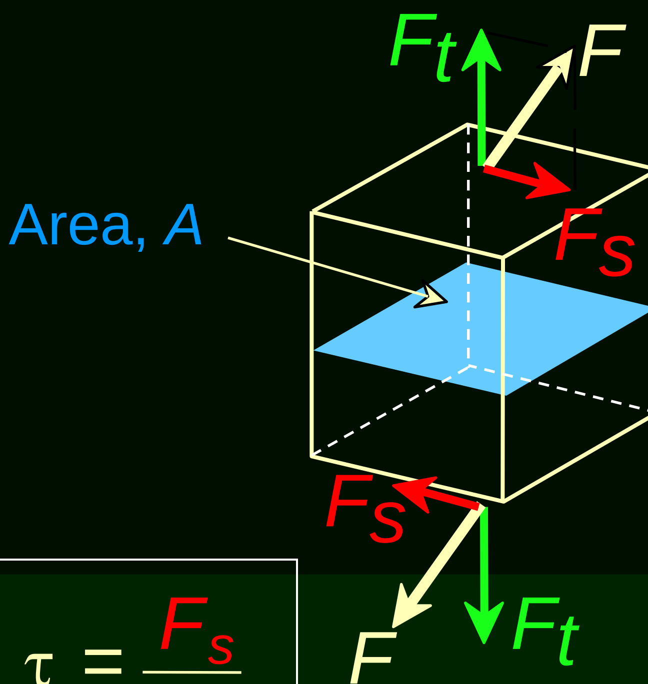
- Tensile stress, σ:



$$\sigma = \frac{F_t}{A_o} = \text{Pa} \;\text{ or }\; \frac{\text{N}}{\text{m}^2}$$

original area
before loading

- Shear stress, τ:



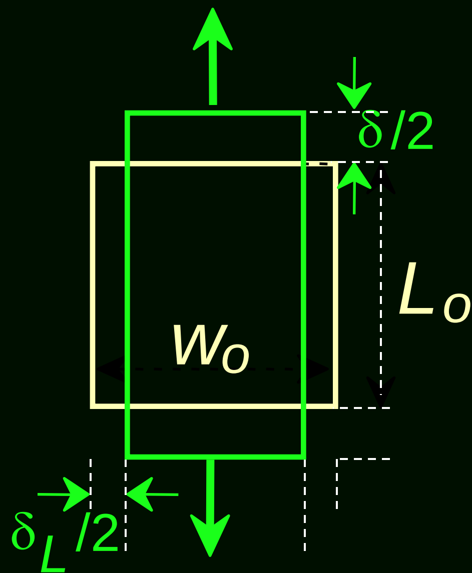
$$\tau = \frac{F_s}{A_o}$$

∴ Stress has units:
N/m^2 or Pa

Engineering Strain

- **Tensile strain:**

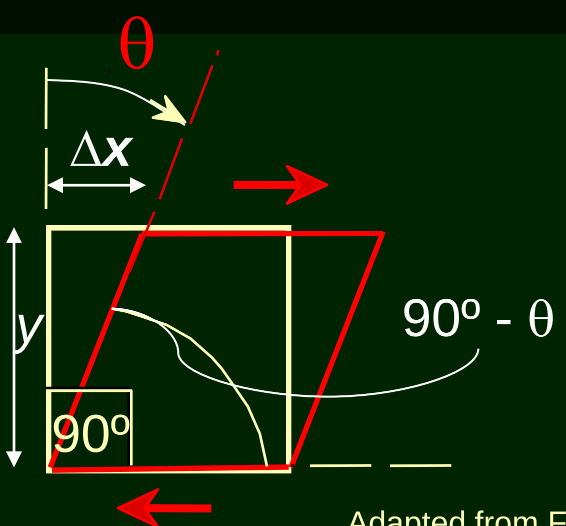
$$\varepsilon = \frac{\delta}{L_o}$$



- **Lateral strain:**

$$\varepsilon_L = \frac{-\delta_L}{w_o}$$

- **Shear strain:**



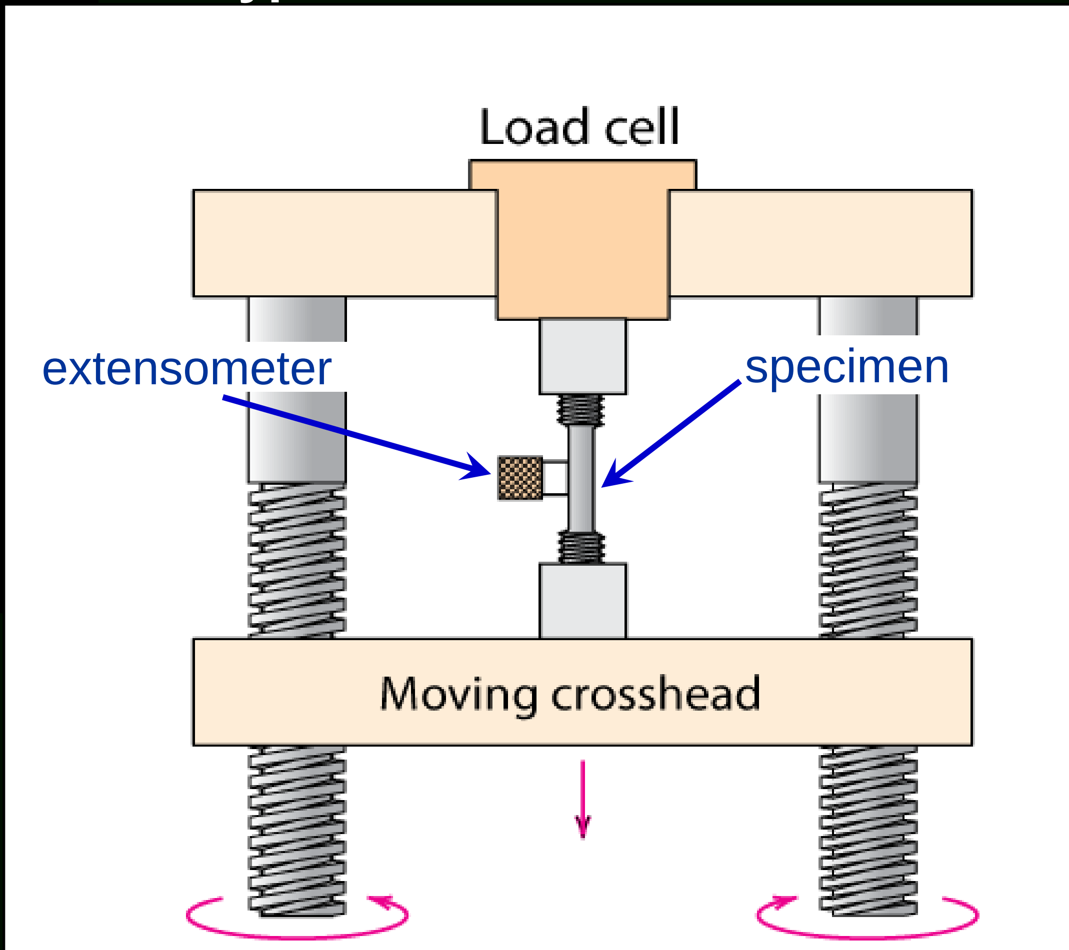
$$\gamma = \Delta x / y = \tan\theta$$

Strain is always dimensionless.

Adapted from Fig. 6.1 (a) and (c), *Callister 7e.*

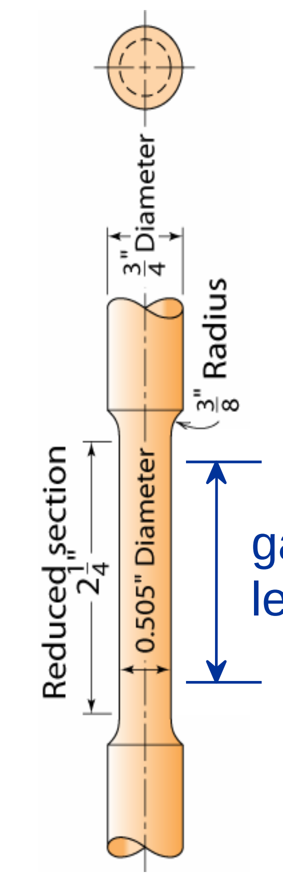
Stress-Strain Testing

- Typical tensile test



Adapted from Fig. 6.3, *Callister 7e.* (Fig. 6.3 is taken from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, p. 2, John Wiley and Sons, New York, 1965.)

- Typical tensile specimen



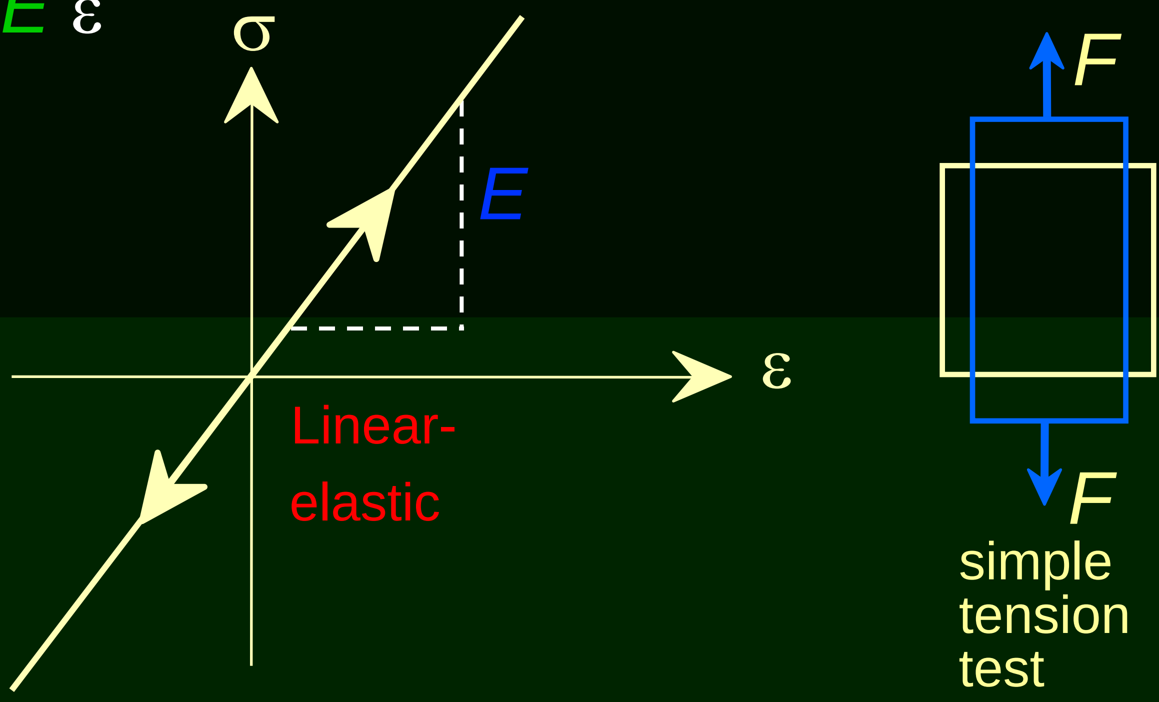
Adapted from Fig. 6.2, *Callister 7e.*

Linear Elastic Properties

- **Modulus of Elasticity, *E*:**
  (also known as Young's modulus)

- **Hooke's Law:**

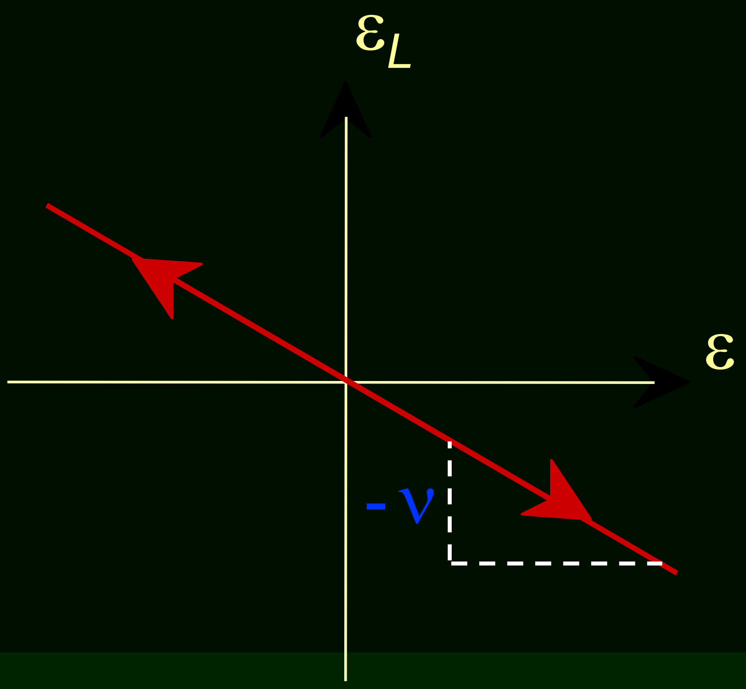
$$\sigma = E\,\varepsilon$$



Poisson's ratio, ν

- **Poisson's ratio, n:**

$$\nu = -\frac{\varepsilon_L}{\varepsilon}$$



metals: ν ~ 0.33
ceramics: ν ~ 0.25
polymers: ν ~ 0.40

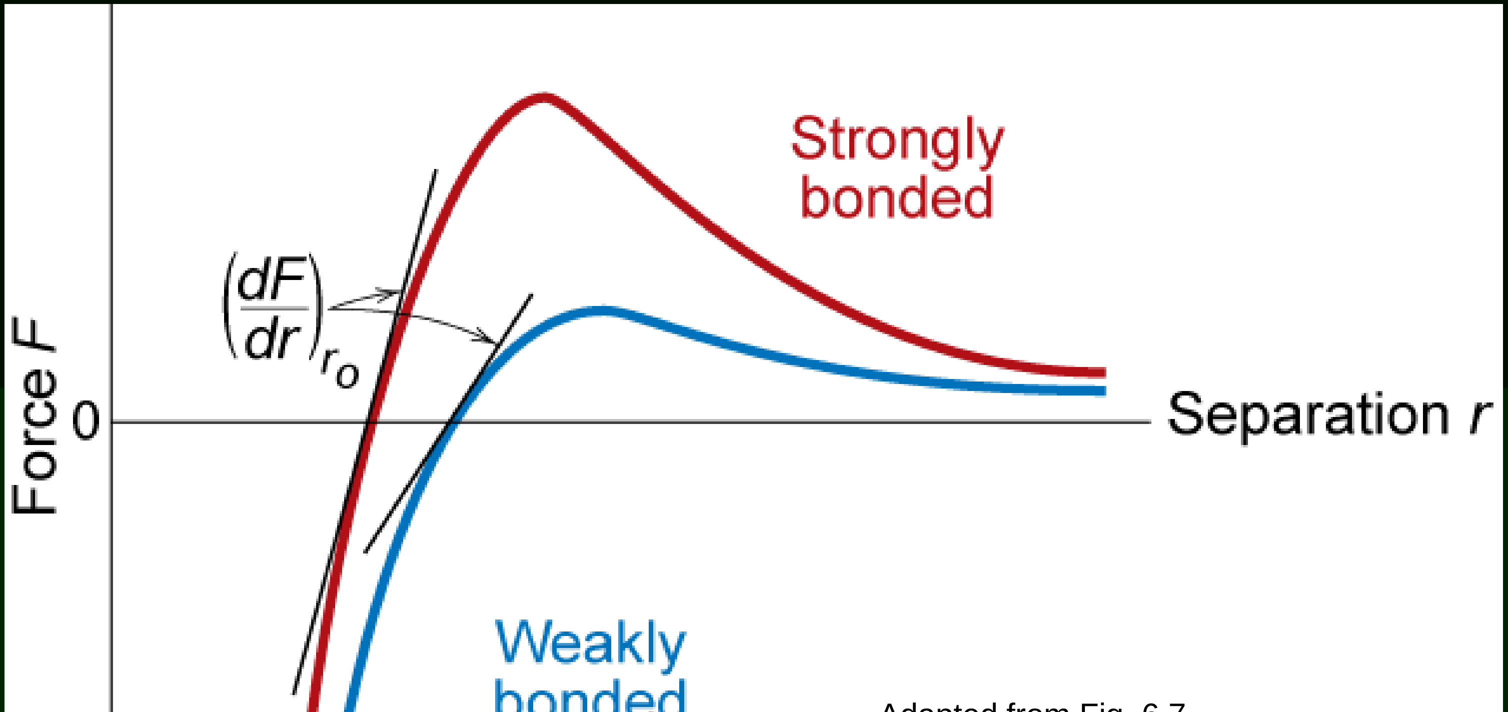
Units:
E: [GPa] or [psi]
ν: dimensionless

$\nu > 0.50$ density increases

$\nu < 0.50$ density decreases (voids form)

Mechanical Properties

- Slope of stress-strain plot (which is proportional to the elastic modulus) depends on bond strength in metals



Adapted from Fig. 6.7, *Callister 7e.*

Anisotropy – Crystals and Textures

Tensor $\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$ or $\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$

For an arbitrary direction

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \bullet \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

Not all components are independent – the tensor is symmetric!

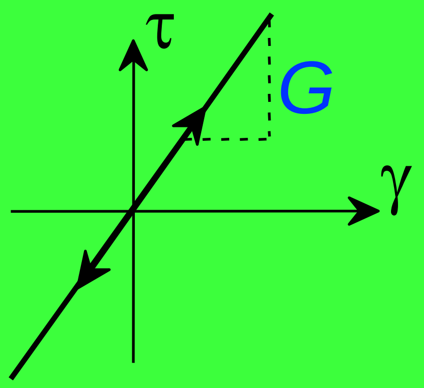
$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

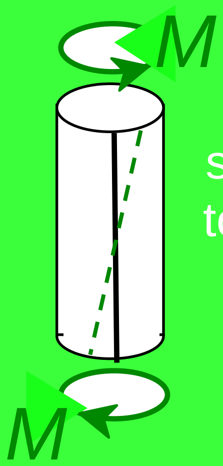
Just for an idea – detailed understanding is not required…

Other Elastic Properties

- **Elastic Shear modulus, *G*:**

$$\tau = G\,\gamma$$

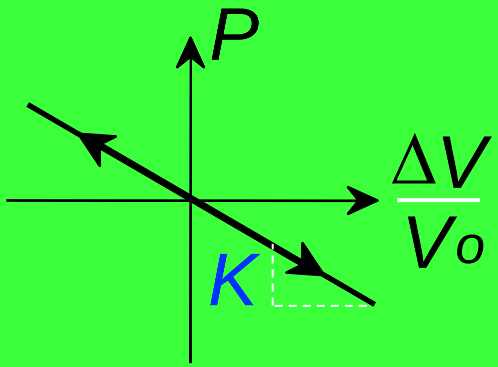


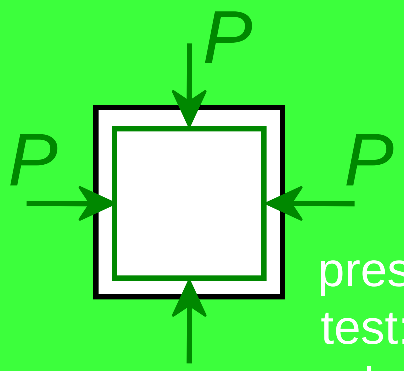


simple torsion test

- **Elastic Bulk modulus, *K*:**

$$P = -K\frac{\Delta V}{V_o}$$



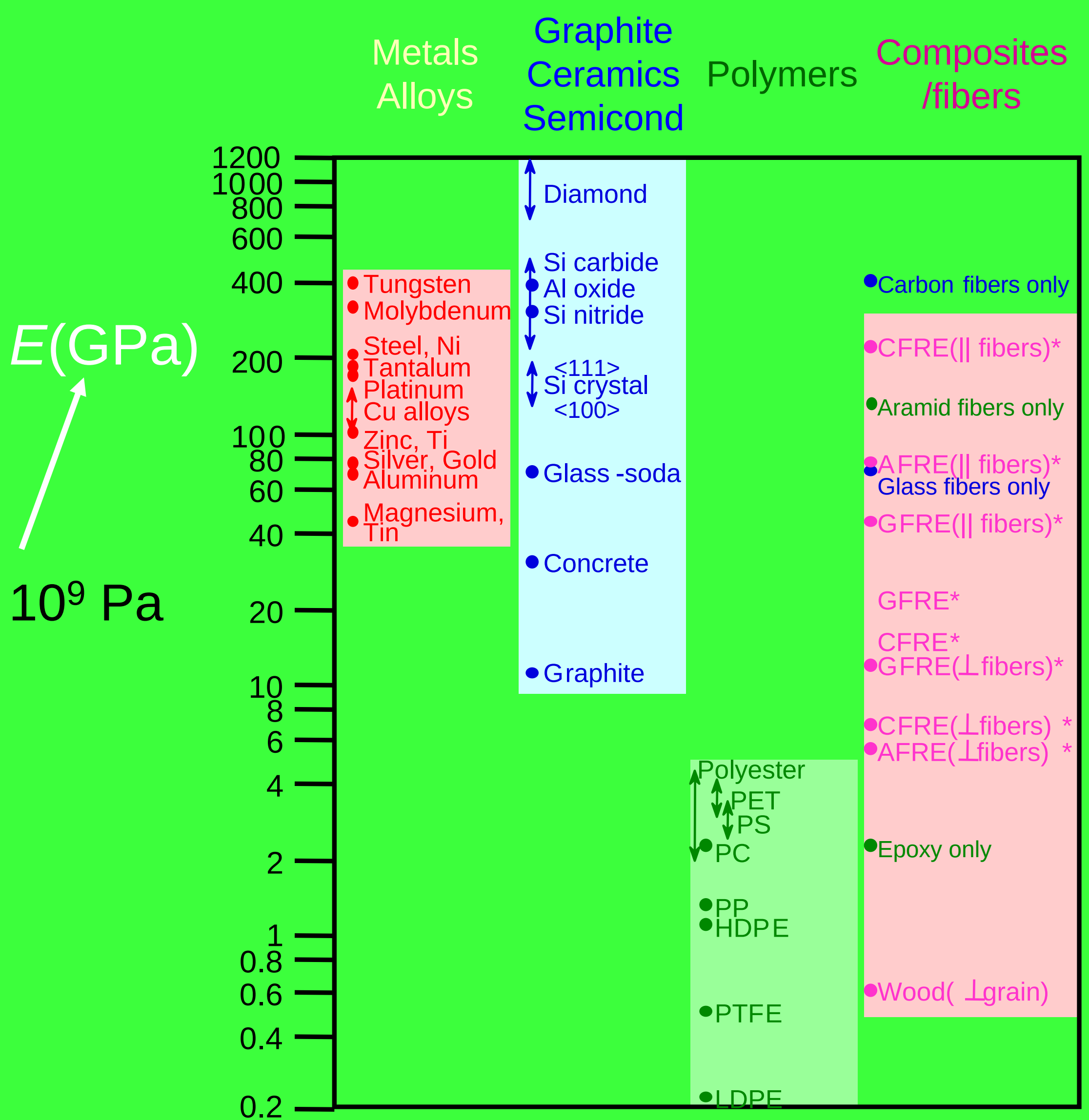


pressure test: Init. vol $= V_o$. Vol chg. $= \Delta V$

- **Special relations for isotropic materials:**

$$G = \frac{E}{2(1+\nu)} \qquad K = \frac{E}{3(1-2\nu)}$$

Young's Moduli - Comparison



Based on data in Table B2, *Callister 7e*.

Composite data based on reinforced epoxy with 60 vol% of aligned carbon (CFRE), aramid (AFRE), or glass (GFRE) fibers.

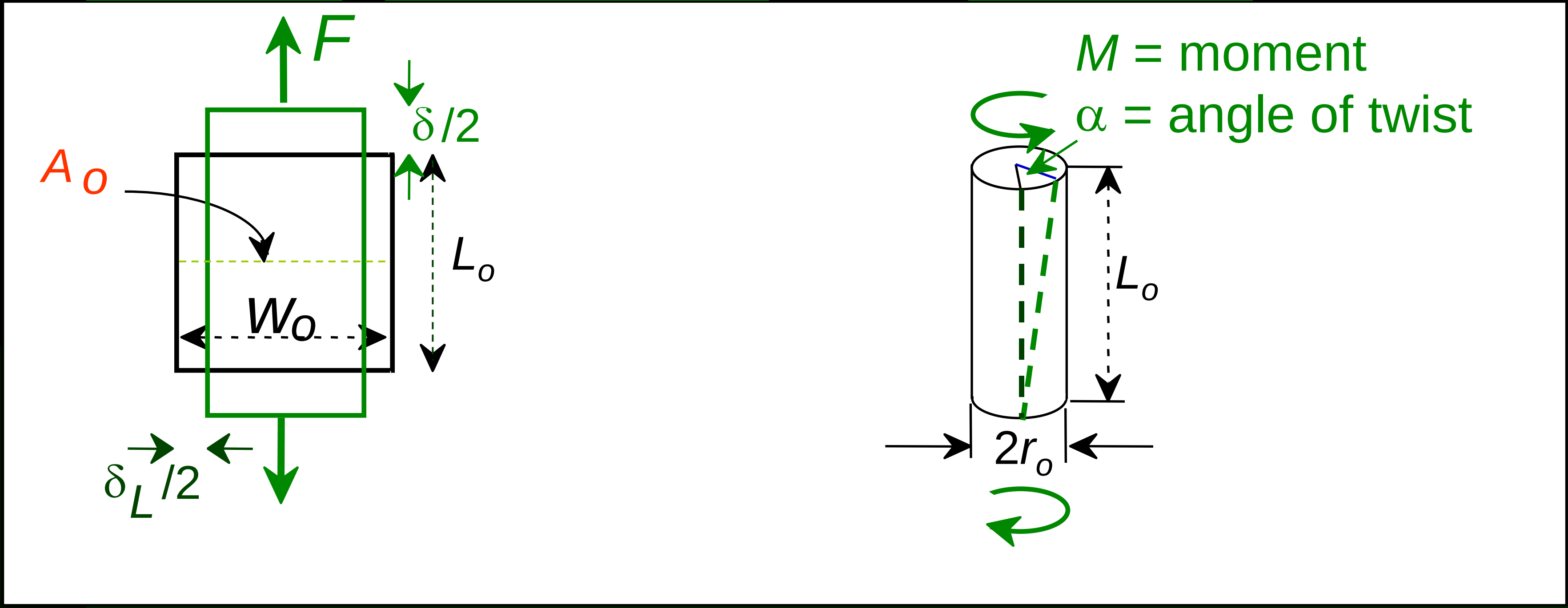
Useful Linear Elastic Relationships

- **Simple tension:**

$$\delta = \frac{F L_o}{E A_o} \qquad \delta_L = -\nu \frac{F w_o}{E A_o}$$

- **Simple torsion:**

$$\alpha = \frac{2 M L_o}{\pi r_o^4 G}$$

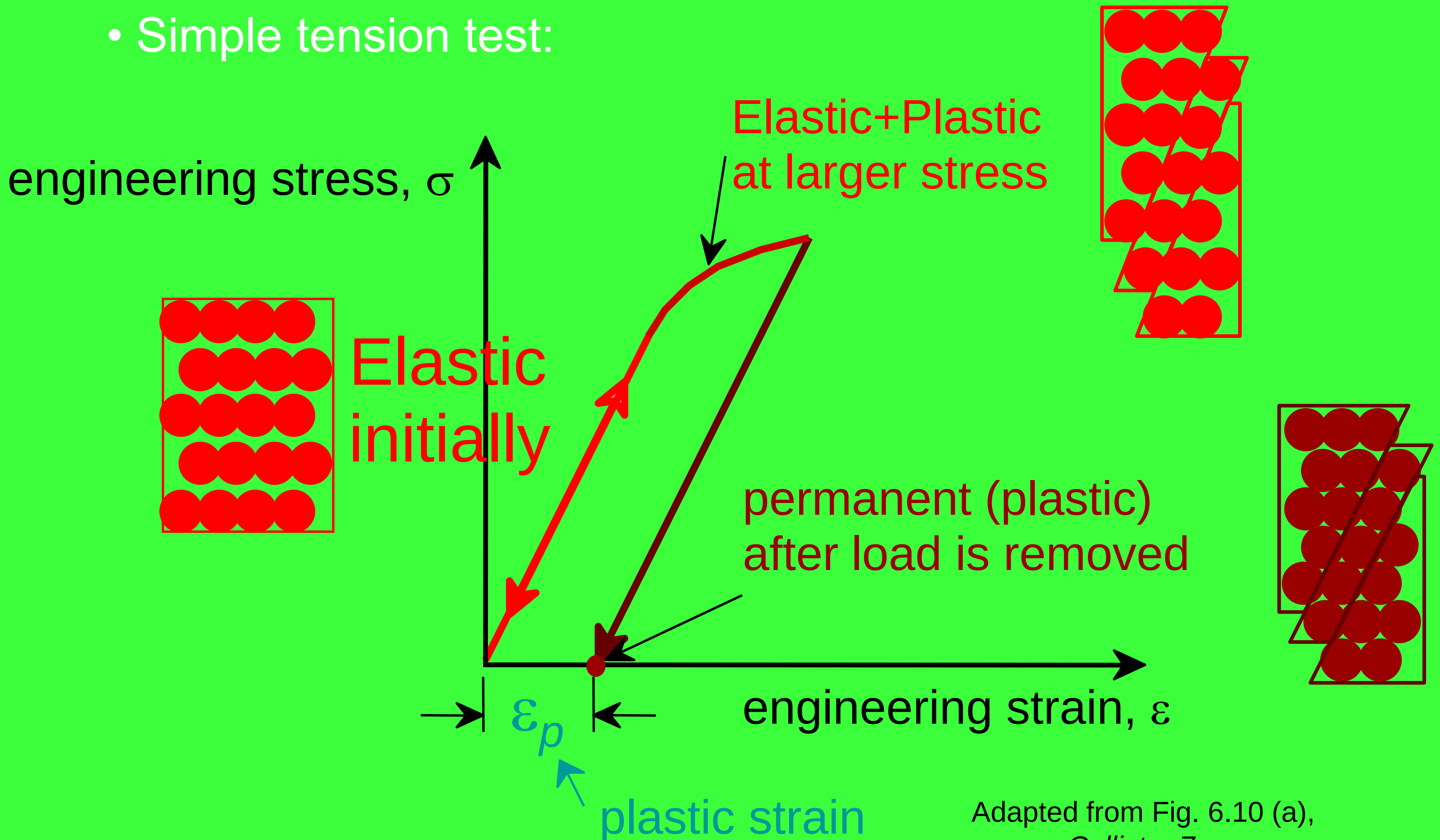


- Material, geometric, and loading parameters all contribute to deflection.
- Larger elastic moduli minimize elastic deflection.

Plastic (Permanent) Deformation

(at lower temperatures, i.e. $T < T_{melt}/3$)

- Simple tension test:

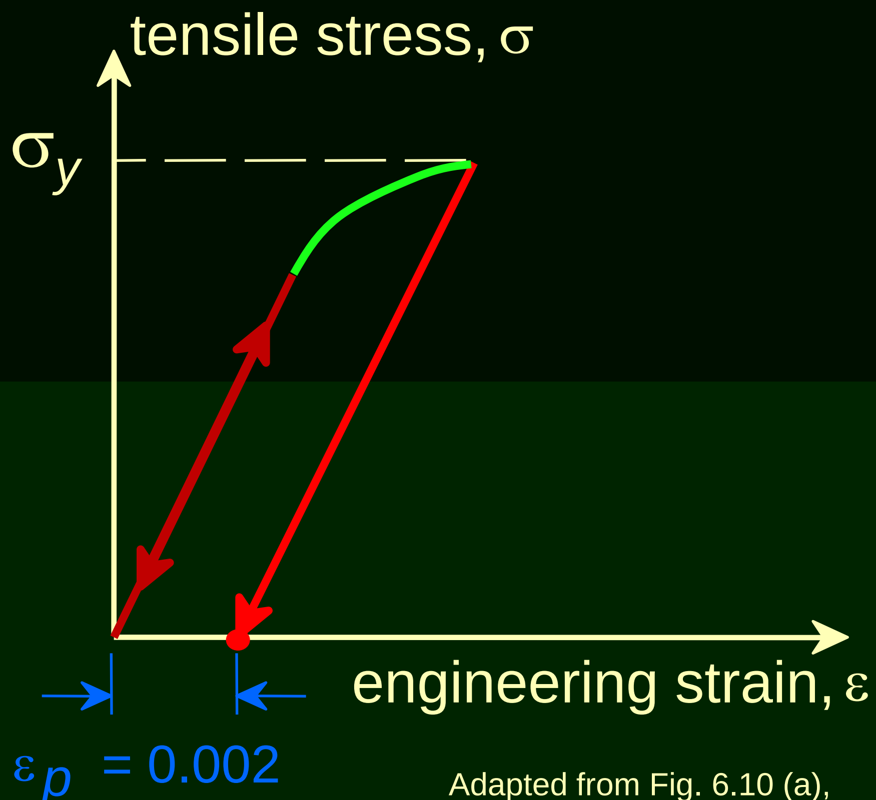


Adapted from Fig. 6.10 (a), *Callister 7e.*

Yield Strength, σ_y

- **Stress at which *noticeable* plastic deformation has occurred.**

when $\varepsilon_p = 0.002$



Adapted from Fig. 6.10 (a), *Callister 7e.*

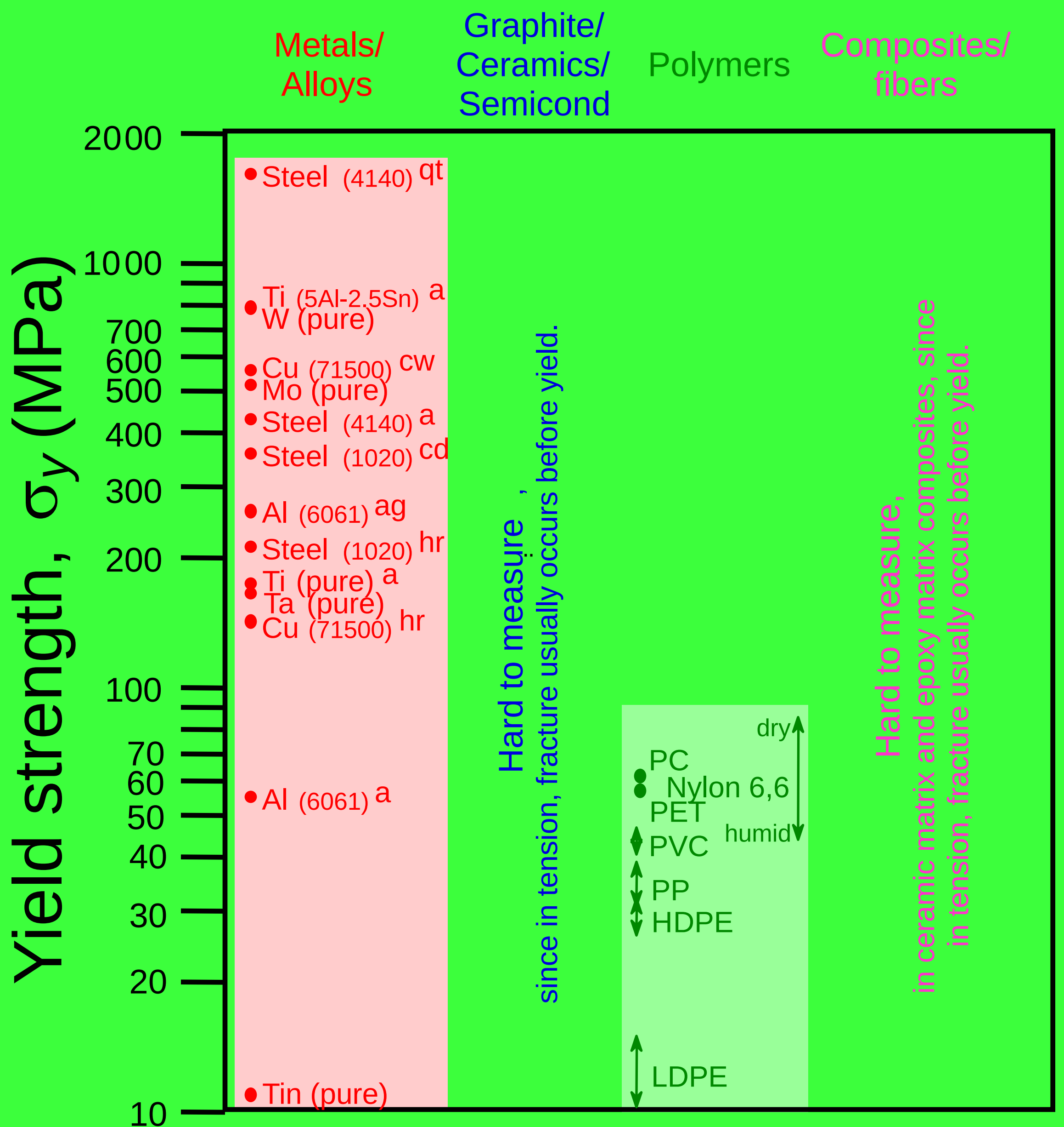
σ_y = yield strength

Note: for 2 inch sample

$\varepsilon = 0.002 = \Delta z/z$

$\therefore\ \Delta z = 0.004$ in or about 100 μm

Yield Strengths - Comparison



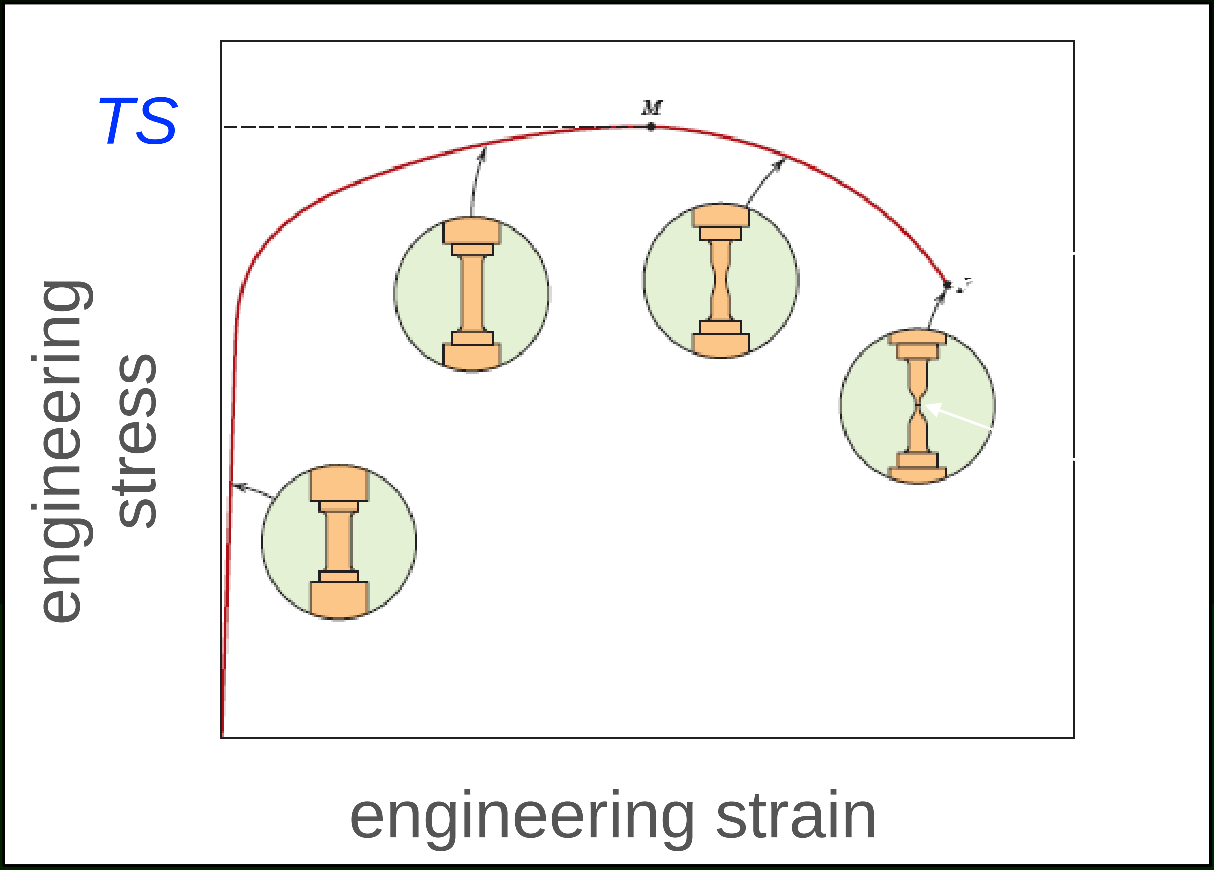
Room T values

Based on data in Table B4, *Callister 7e*.

a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

Tensile Strength, TS

- Maximum stress on engineering stress-strain curve.



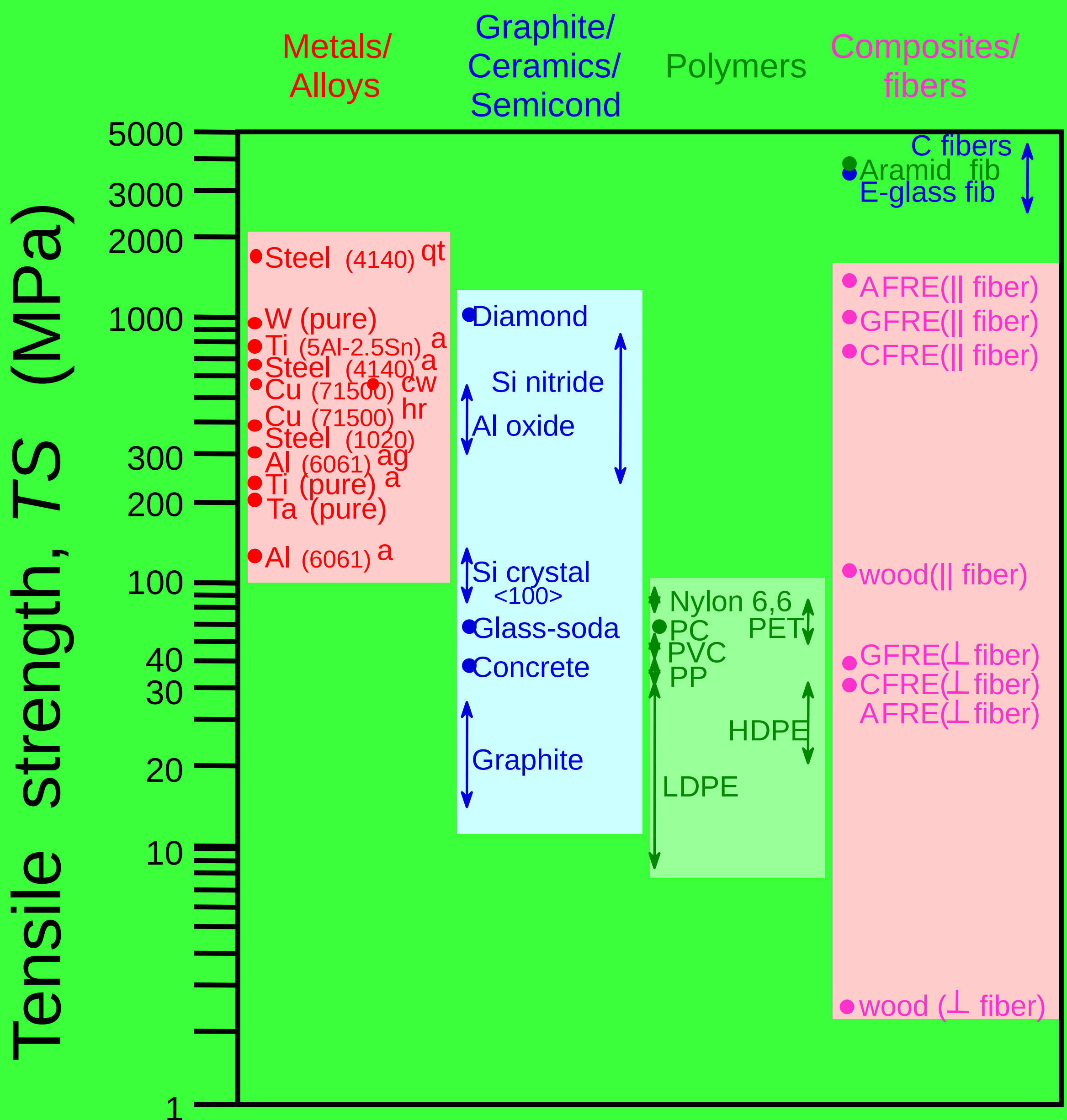
Adapted from Fig. 6.11, *Callister 7e.*

F = fracture or ultimate strength

Neck – acts as stress concentrator

- Metals: occurs when noticeable necking starts.
- Polymers: occurs when polymer backbone chains are aligned and about to break.

Tensile Strengths - Comparison



Room Temp. values

Based on data in Table B4, *Callister 7e*.

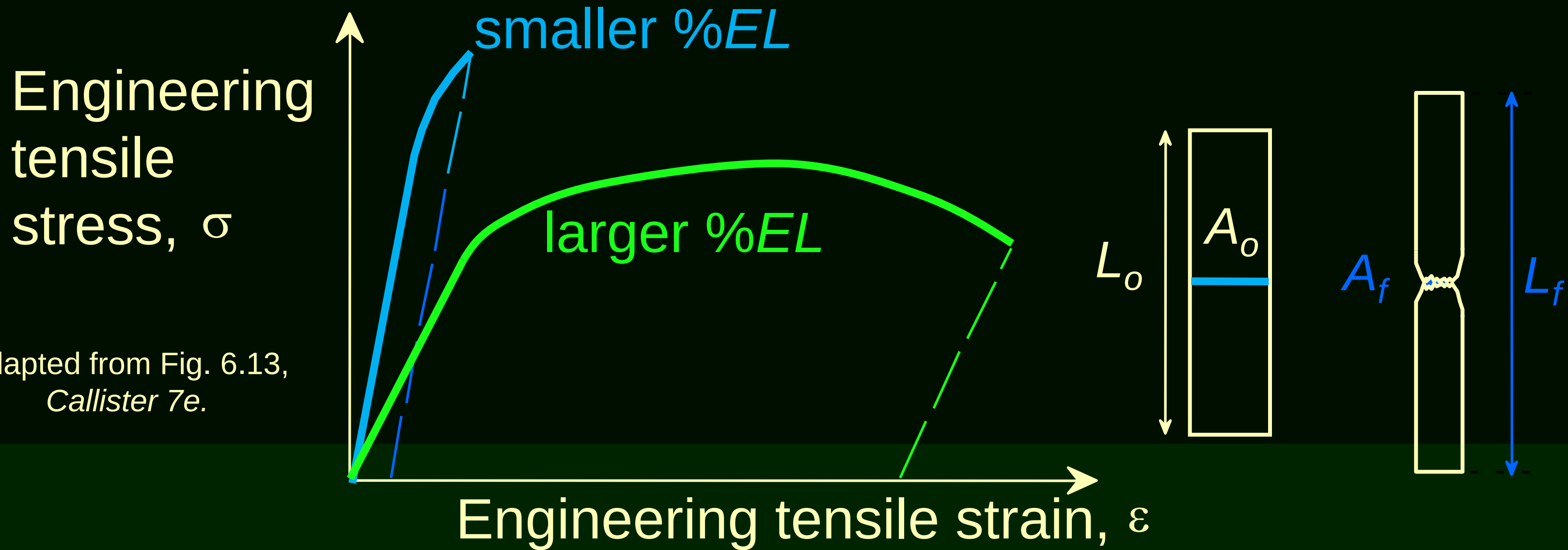
a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

AFRE, GFRE, & CFRE = aramid, glass, & carbon fiber-reinforced epoxy composites, with 60 vol% fibers.

Ductility

- Plastic tensile strain at failure:

$$\%EL = \frac{L_f - L_o}{L_o} \times 100$$



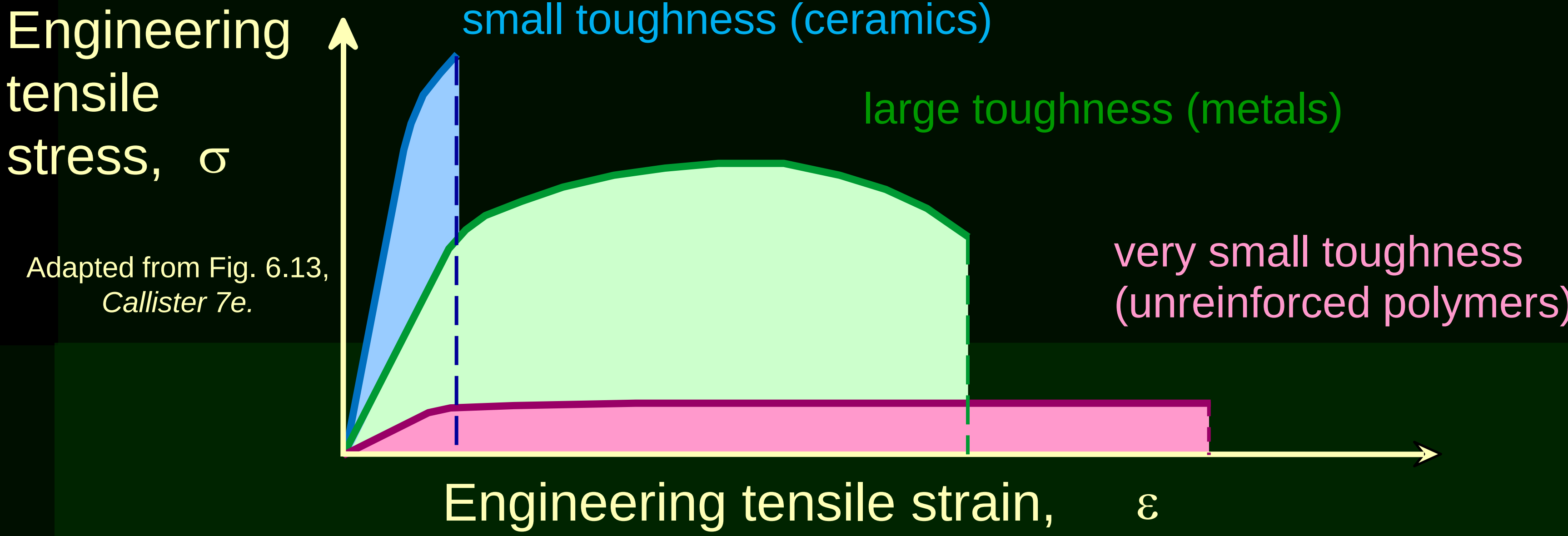
Adapted from Fig. 6.13, *Callister 7e.*

- Another ductility measure:

$$\%RA = \frac{A_o - A_f}{A_o} \times 100$$

Toughness

- Energy to break a unit volume of material
- Approximate by the area under the stress-strain curve.

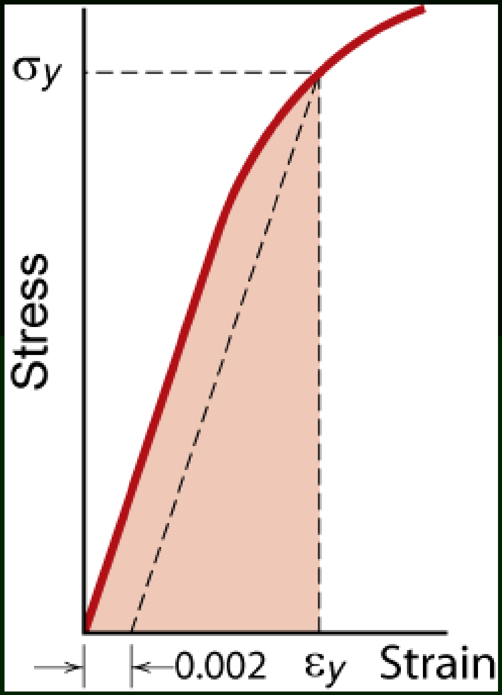


Adapted from Fig. 6.13, *Callister 7e.*

Brittle fracture: elastic energy
Ductile fracture: elastic + plastic energy

Resilience, U_r

- Ability of a material to store energy
  - Energy stored best in elastic region



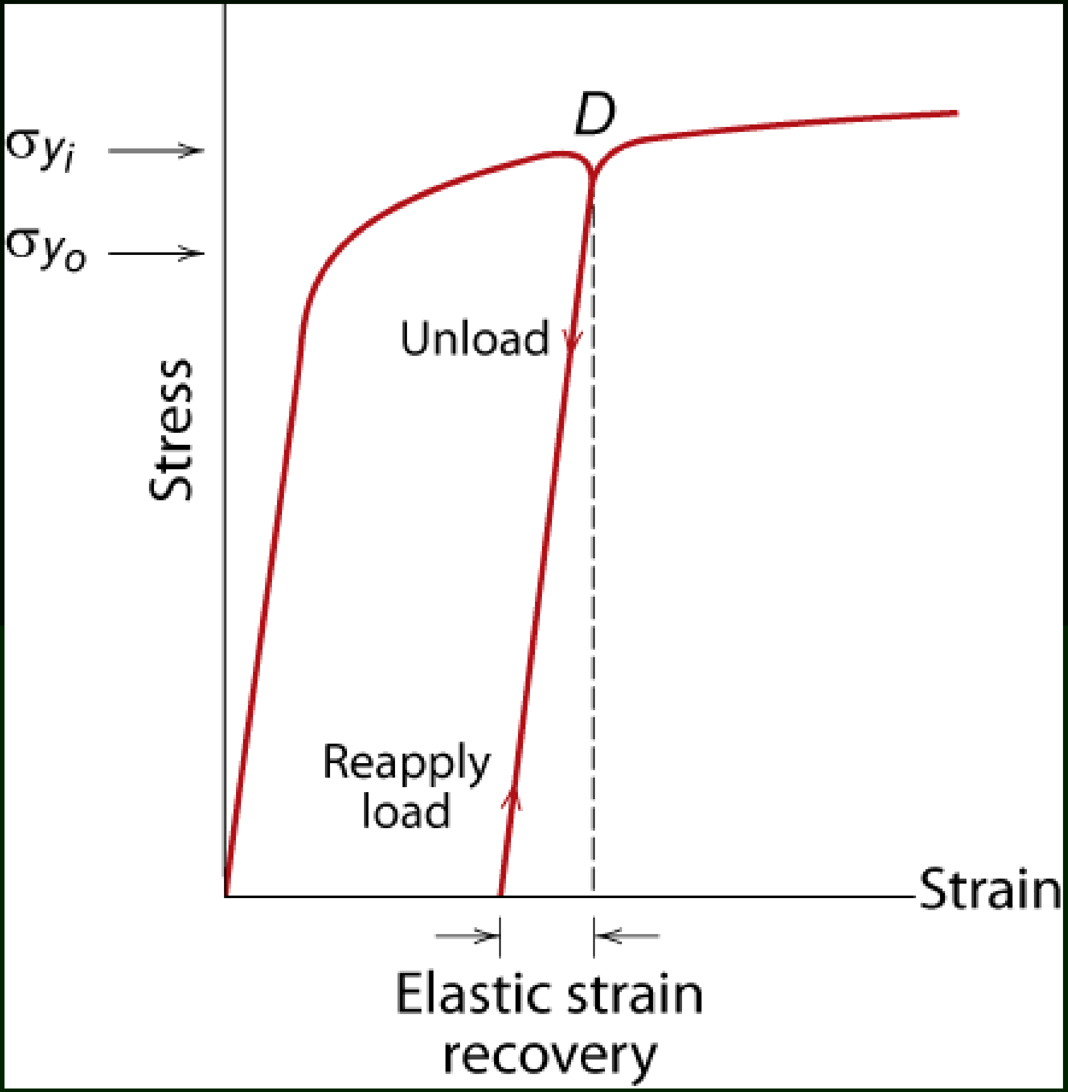
Adapted from Fig. 6.15, *Callister 7e.*

$$U_r = \int_0^{\varepsilon_y} \sigma \, d\varepsilon$$

If we assume a linear stress-strain curve this simplifies to:

$$U_r \cong \frac{1}{2}\sigma_y e_y$$

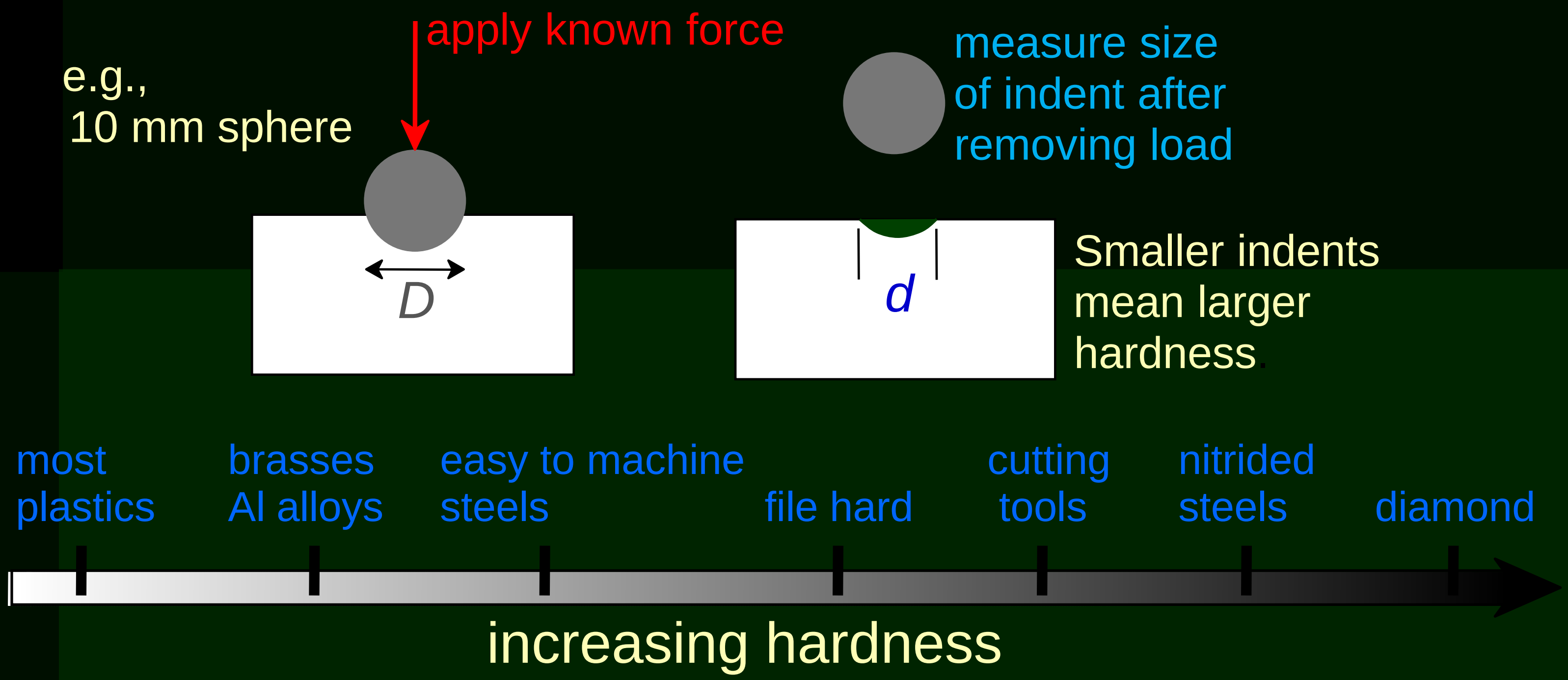
Elastic Strain Recovery



Adapted from Fig. 6.17,
Callister 7e.

Hardness

- Resistance to permanently indenting the surface.
- Large hardness means:
  - -resistance to plastic deformation or cracking in compression.
  - -better wear properties.



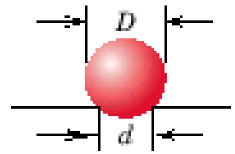
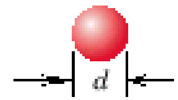
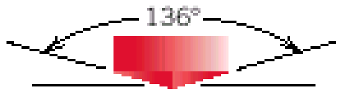
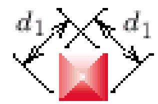
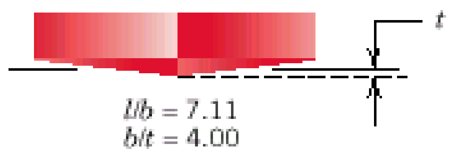
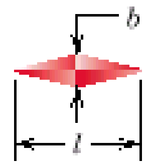
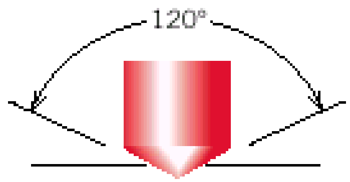
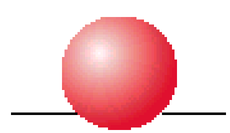
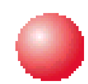
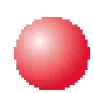
Hardness Measures

- Rockwell
  - No major sample damage
  - Each scale runs to 130 but only useful in range 20-100.
  - Minor load   10 kg
  - Major load   60 (A), 100 (B) & 150 (C) kg
    - A = diamond, B = 1/16 in. ball, C = diamond

- HB = Brinell Hardness
  - *TS* (psia) = 500 x HB
  - *TS* (MPa) = 3.45 x HB

Hardness Measurements

Table 6.4 Hardness Testing Techniques

Test	Indenter	Shape of Indentation: Side View	Shape of Indentation: Top View	Load	Formula for Hardness Number[a]
Brinell	10-mm sphere of steel or tungsten carbide			P	$\text{HB} = \dfrac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid			P	$\text{HV} = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid			P	$\text{HK} = 14.2P/l^2$
Rockwell and Superficial Rockwell	{ Diamond cone $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$ in. diameter steel spheres	 	 	60 kg, 100 kg, 150 kg } Rockwell 15 kg, 30 kg, 45 kg } Superficial Rockwell	

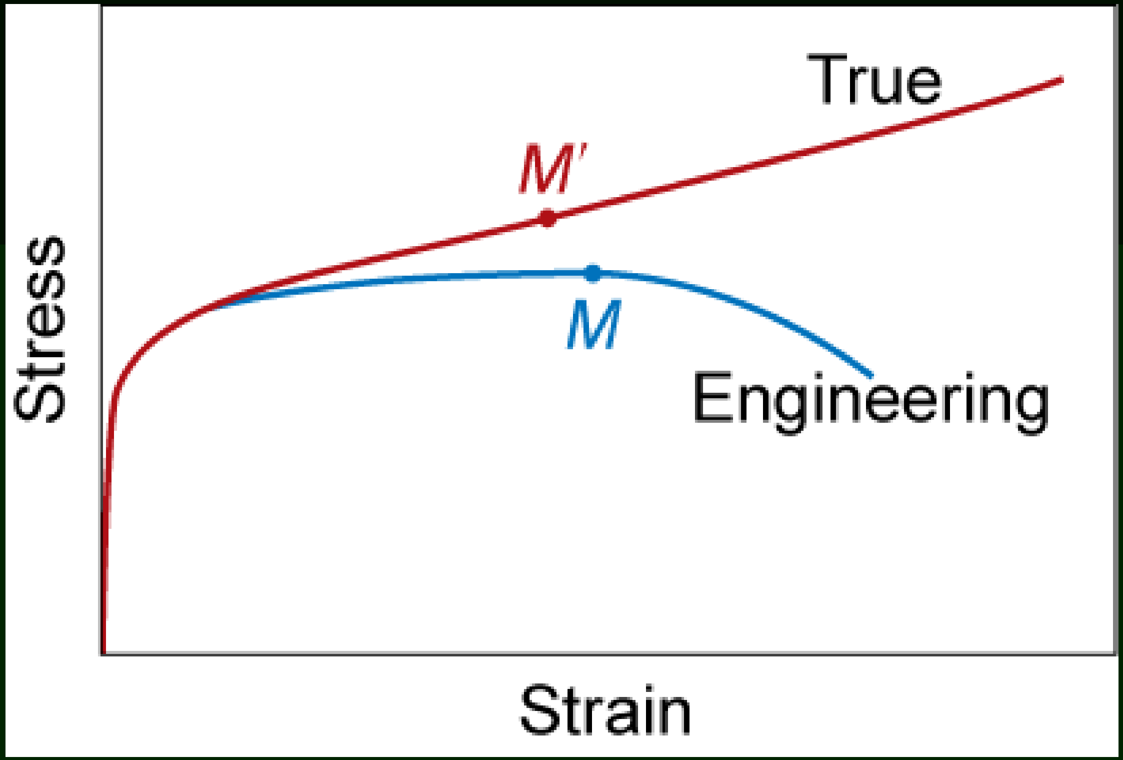
[a] For the hardness formulas given, P (the applied load) is in kg, while D, d, d_1, and l are all in mm.

True Stress and Strain

Note: S.A. changes when sample stretched

- True stress $\sigma_T = F/A_i$
- True Strain $\varepsilon_T = \ln\left(\ell_i/\ell_o\right)$

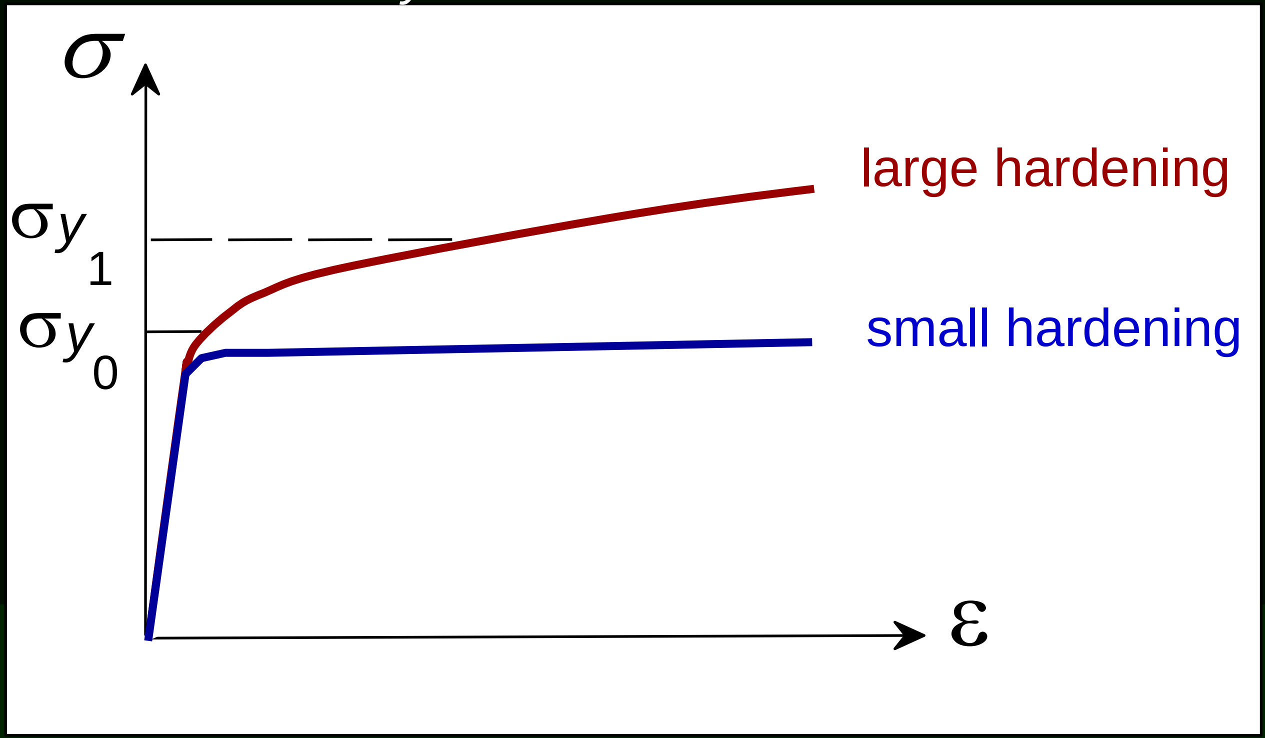
$$\sigma_T = \sigma\left(1+\varepsilon\right)$$
$$\varepsilon_T = \ln\left(1+\varepsilon\right)$$



Adapted from Fig. 6.16, *Callister 7e.*

Hardening

- An increase in σ_y due to plastic deformation.



$$\sigma_T = K(\varepsilon_T)^n$$

hardening exponent:
n = 0.15 (some steels)
to n = 0.5 (some coppers)

"true" stress (F/A)

"true" strain: $\ln(L/L_o)$

Design or Safety Factors

- Design uncertainties mean we do not push the limit.
- Factor of safety, N

$$\sigma_{working} = \frac{\sigma_y}{N}$$

Often N is between 1.2 and 4

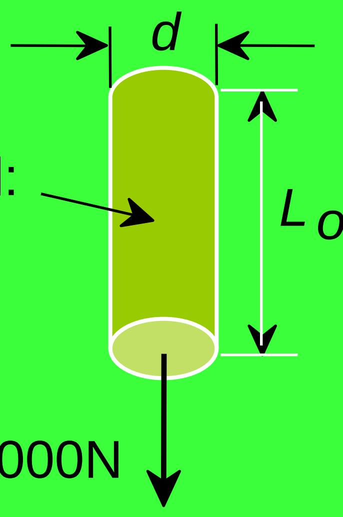
- Example: Calculate a diameter, d, to ensure that yield does not occur in the 1045 carbon steel rod below. Use a factor of safety of 5.

$$\sigma_{working} = \frac{\sigma_y}{N}$$

$$\frac{220{,}000N}{\pi\left(d^2/4\right)}$$

5

1045 plain carbon steel:
σ_y = 310 MPa
TS = 565 MPa



F = 220,000N

d = 0.067 m = 6.7 cm