MA 2326

Assignment 7

Due never

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1. A class of popular models for the dynamics of the populations of two species is

$$x'(t) = ax(t)^2 + bx(t)y(t) + cx(t), \quad y'(t) = dx(t)y(t) + ey(t)^2 + fy(t).$$

(a) What are the equilibria?

(b) Find the linearisation about each equilibrium.
The linearisation about (ξ, η) is

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = A \begin{pmatrix} x - \xi \\ y - \eta \end{pmatrix},$$

where

$$A = \begin{pmatrix} 2a\xi + b\eta + c & b\xi \\ d\eta & d\xi + 2e\eta + c \end{pmatrix}.$$

Substituting the four equilibria we found above gives

$$A = \begin{pmatrix} c & 0 \\ 0 & f \end{pmatrix}$$

or

$$A = \begin{pmatrix} c - bf/e & 0 \\ -df/e & -f \end{pmatrix}$$

or

$$A = \begin{pmatrix} -c & -bc/a \\ 0 & f - cd/a \end{pmatrix}$$

or

$$A = \frac{1}{ae - bd} \begin{pmatrix} abf - ace & b^2f - bce \\ -adf + cd^2 & -aef + cde \end{pmatrix}.$$

(c) Determine, where possible, the stability properties of each equilibrium.

2. The time reversed Van der Pol equation is, after reduction of order,

$$x'(t) = y(t), \quad y'(t) = -x(t) + \mu(x(t)^2 - 1)y(t),$$

where μ is a positive parameter. $(0, 0)$ is equilibrium. Show that this equilibrium is strictly stable by finding a strict Lyapunov function.