MA 2326
Assignment 4
Due 17 February 2015

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1. Compute $\exp(xA)$ for
$$A = \begin{pmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix}$$
Note: The matrix A is real, so $\exp(xA)$ will be real as well. You should simplify your answer sufficiently that this is obvious.

2. (a) An $n \times n$ matrix M if of rank k if and only if there is an $n \times k$ matrix P and a $k \times n$ matrix Q such that
$$M = PQ$$
and QP is invertible. Prove that in this case[1]
$$\exp(M) = I + P(QP)^{-1}\left(\exp(QP) - I\right)Q.$$
Note: The equation is always correct, but is only of practical use if k is much smaller than n. Do not attempt to use the equation $(QP)^{-1} = P^{-1}Q^{-1}$. This always fails if $k < n$, because neither P nor Q can be invertible in that case.

[1] The two I's on the right hand side of this equation are not equal! The first occurence of I is the $n \times n$ identity matrix and the second occurence of I is the $k \times k$ identity matrix.

(b) Use the preceding identity to compute $\exp(xA)$ where

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

Note: You may use the identity from the previous part even if you didn't succeed in proving it.