MA 2326
Assignment 3
Due 10 February 2015

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1. In all problems on this assignment A is a continuous $n \times n$ matrix valued function defined on an interval $J \subset \mathbf{R}$, b is a continuous $n \times 1$ matrix valued function, *i.e.* a column vector valued function, on the same interval, and W is a continuously differentiable $n \times n$ matrix valued function on the rectangle $J \times J \subset \mathbf{R}^2$. The partial derivatives with respect to its first and second arguments are therefore continuous $n \times n$ matrix valued functions on $J \times J$. To avoid getting them confused we will write W' for the derivative with respect to the first argument and $\dot{W}$ for the derivative with respect to the second argument. We assume

$$W'(s,t) = A(s)W(s,t), \quad \dot{W}(s,t) = -W(s,t)A(t), \quad W(s,s) = I$$

for all $s, t \in \mathbf{R}$, where I is the $n \times n$ identity matrix. Prove that

(a)

$$W(r,s)W(s,r) = I$$

for all $r, s \in \mathbf{R}$.

(b)

$$W(r,s)W(s,t) = W(r,t)$$

for all $r, s, t \in \mathbf{R}$.

2. Show that if the vector valued function y on J is defined by

$$y(x) = W(x, x_0)y_0 + \int_{x_0}^{x} W(x, z)b(z)\,dz$$

where $x_0 \in J$ and y_0 is a (constant) column vector then

$$y'(x) = A(x)y(x) + b(x), \quad y(x_0) = y_0.$$

3. Show that if y is a continuously differentiable vector valued function on J satisfying

$$y'(x) = A(x)y(x) + b(x), \quad y(x_0) = y_0.$$

then

$$y(x) = W(x, x_0)y_0 + \int_{x_0}^{x} W(x, z)b(z)\,dz.$$

Note: You may use the results of earlier questions even if you didn't succeed in proving them. You may find the quantity

$$u(x) = W(x_0, x)y(x) - \int_{x_0}^{x} W(x_0, z)b(z)\,dz$$

useful.