MA 342H
Assignment 2
Due 5 March 2014

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1. Solve the fully nonlinear first order scalar equation

$$(1 + x^2 + y^2)^2(u_x^2 + u_y^2) = 1$$

with initial conditions

$$u(x, 0) = 0.$$

There are two solutions. Take the one where $u_y(x, 0) > 0$.
Note: The characteristic equations can be solved explicitly, and then the variables s and t can be eliminated, but this is far from easy. Feel free to stop at the point where you have written down the characteristic system and appropriate initial conditions.

2. Find the Euler-Lagrange equation for the Lagrangian

$$L(x, u, u_x, u_{xx}) = uu_x - u_x u_{xx}.$$

Simplify as much as possible. What is going on here?

3. As discussed in lecture, the minimal surface problem, with Lagrangian

$$L(x, y, z, z_x, z_y) = \sqrt{1 + z_x^2 + z_y^2}$$

has six independent symmetries, the three translations

$$(x, y, z) \mapsto (x + s, y, z),$$

$$(x, y, z) \mapsto (x, y + s, z),$$
$$(x, y, z) \mapsto (x, y, z + s),$$

and the three rotations

$$(x, y, z) \mapsto (x \cos\theta - y \sin\theta, x \sin\theta + y \cos\theta, z),$$
$$(x, y, z) \mapsto (x \cos\theta - z \sin\theta, y, x \sin\theta + z \cos\theta),$$
$$(x, y, z) \mapsto (x, y \cos\theta - z \sin\theta, y \sin\theta + z \cos\theta).$$

For each of these, find the infinitesimal generator and the corresponding conserved current.