MA 2326
Assignment 2
Due 13 February 2014

`Id: 2326-s2014-2.m4,v 1.1 2014/02/06 19:55:03 john Exp john`

1. Solve the initial value problem

$$dy/dx = x + 2y + 3, \quad y(4) = 5.$$

Solution: Applying the general solution formula

$$y(x) = y_0 \exp\left(\int_{x_0}^{x} a(s)\,ds\right) + \int_{x_0}^{x} \exp\left(\int_{s}^{x} a(r)\,dr\right) b(s)\,ds$$

with

$$x_0 = 4, \quad y_0 = 5, \quad a(x) = 2, \quad b(x) = x + 3$$

we get

$$y(x) = 5\exp\left(\int_{4}^{x} 2\,ds\right) + \int_{4}^{x} \exp\left(\int_{s}^{x} 2\,dr\right)(s+3)\,ds.$$

The solution is therefore

$$y(x) = \frac{43e^{2x-8} - 2x - 7}{4}.$$

2. Solve the initial value problem

$$x'(t) = t^{-1}x(t) + 1, \quad x(1) = 0.$$

Solution: Applying the general solution formula

$$x(t) = x_0 \exp\left(\int_{t_0}^{t} a(s)\,ds\right) + \int_{t_0}^{t} \exp\left(\int_{s}^{t} a(r)\,dr\right) b(s)\,ds$$

with $t_0 = 1$, $x_0 = 0$, $a(t) = t^{-1}$, $b(t) = 1$ gives

$$x(t) = \int_{1}^{t} \exp\left(\int_{s}^{t} r^{-1}\,dr\right)\,ds$$

or

$$x(t) = t\log t.$$