MA 2326
Assignment 6
Due 3 April 2014

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1. The equilibria of the autonomous system
$$x' = 2x + y - 2x^3 \quad y' = 2x - y - 2x^3$$
are $(-1, 0)$, $(0, 0)$ and $(1, 0)$. Show that
$$V(x, y) = x^4 - 2x^2 + y^2$$
is a strict Lyapunov function for the equilibria $(-1, 0)$ and $(1, 0)$.

2. For which of the following is the origin a stable equilibrium? For which is it strictly stable?

   (a)
$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

   (b)
$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} -1 & 3 & -2 \\ -3 & -1 & 1 \\ 2 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

   (c)
$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} -9 & 12 & 0 \\ 12 & -16 & 0 \\ 20 & 15 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$