MA 2326
Assignment 5
Due 20 March 2014

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1. Find the equilibria of the autonomous system
$$x'(t) = x(x^3 - 2y^3), \quad y' = y(2x^3 - y^3).$$

2. Suppose that U and V are open subsets of $\mathbf{R}^m$ and that $\varphi: U \to V$ and $\psi: V \to U$ are continuously differentiable and that ψ is the inverse of φ. Suppose also that $F: U \to \mathbf{R}^m$ and that $G: V \to \mathbf{R}^m$ is defined by
$$G_j(y) = \sum_{k=1}^{m} \frac{\partial \varphi_j}{\partial x_k}(\psi(y)) F_k(\psi(y)).$$

   (a) Prove that $x: I \to U$ is a solution to the autonomous system
   $$x'(t) = F(x(t))$$
   if and only if $y: I \to V$, defined by
   $$y(t) = \varphi(x(t)),$$
   is a solution to the autonomous system
   $$y'(t) = G(y(t)).$$
   I is an interval.

   (b) Prove that ξ is an equilibrium of $x'(t) = F(x(t))$ if and only if $\eta = \varphi(\xi)$ is an equilibrium of $y'(t) = G(y(t))$.

   (c) Prove that ξ is a stable equilibrium if and only if η is.

   (d) Prove that ξ is a strictly stable equilibrium if and only if η is.