MA 2326
Assignment 4
Due 6 March 2014

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1. Given that
$$x_1(t) = t^2$$
and
$$x_2(t) = t^3$$
are linearly independent solutions of the differential equation
$$t^2 x''(t) - 4tx'(t) + 6x(t) = 0$$
in the interval $(0, \infty)$, find the fundamental matrix for
$$A(t) = \begin{pmatrix} 0 & 1 \\ -6t^{-2} & 4t^{-1} \end{pmatrix}.$$

2. Given that
$$x(t) = t$$
is a solution to the differential equation
$$t^2 x''(t) - tx'(t) + x(t) = 0$$
on the interval $(0, \infty)$, find a second solution by Wronskian reduction of order.

3. Given that
$$W(s,t) = \begin{pmatrix} \cos \log(s/t) & -s \sin \log(s/t) \\ t^{-1} \sin \log(s/t) & st^{-1} \cos \log(s/t) \end{pmatrix}$$

is the fundamental matrix for

$$A(t) = \begin{pmatrix} 0 & 1 \\ -t^{-2} & -t^{-1} \end{pmatrix}$$

in the interval $(0, \infty)$, solve the initial value problem

$$x(t_0) = x_0 \quad x'(t_0) = v_0$$

inhomogeneous linear differential equation

$$t^2 x''(t) + t x'(t) + x(t) = f(t).$$