1. (a) What is the solution to the initial value problem

$$u_{tt} - c^2 u_{xx} - c^2 u_{yy} = 0$$

$$u(0,x,y) = f(x,y) \quad u_t(0,x,y) = g(x,y)$$

in $\mathbf{R}^{1+2}$?

   (b) State Young's Inequality, together with its hypotheses.

   (c) Show that, when $f = 0$,

$$\|u(t,\cdot)\|_r \le C t^{-\alpha} \|g\|_q$$

if $\frac{1}{q} - \frac{1}{r} \le \frac{1}{2}$, with constants C and α which depend only on q and r. You should find α, at least, explicitly.

2. (a) How are the following defined for distributions?
      i. Addition
      ii. Differentiation
      iii. Multiplication
      iv. Convolution

   (b) What is meant by a *fundamental solution* of a linear constant coefficient partial differential equation?

   (c) How do you get to the solution to an inhomogeneous linear constant coefficient partial differential equation from a fundamental solution?

3. (a) Find the characteristic ordinary differential equations for the first order linear scalar equation

$$(t-a)u_t + (x-b)u_x = 0$$

   (b) Solve them.

   (c) Find the solution of the initial value problem $u(0,x) = f(x)$.

   (d) Discuss existence and uniqueness of global solutions.

4. (a) What is meant by a *shock* in the context of Burgers' Equation?

   (b) What is a weak solution to Burgers' Equation?

   (c) What is the entropy condition?

   (d) Give an example of a weak solution which does not satisfy the entropy condition.