# MA 3426
# Assignment 3
# Due 19 March 2013

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1. Show that
$$u(x, y) = \frac{1}{4\pi} \log(x^2 + y^2)$$
is a fundamental solution to the Laplace Equation in $\mathbf{R}^2$.
*Hint:* Write $\langle u_{xx} + u_{yy}, \varphi \rangle$ as an integral. Split this integral into three pieces: one where $x^2 + y^2 < r_1^2$, one where $r_1^2 \leq x^2 + y^2 \leq r_2^2$, and one where $r_2^2 < x^2 + y^2$. Apply Green's Second Identity to the second of these, and let $r_1$ and $r_2$ tend to 0 and $\infty$, respectively.

2. Solve the initial value problem $u(t, x) = f(x)$ for the first order linear scalar equation
$$(t^2 + 1)u_t + (1 + x^2)u_x = 0.$$
For which $(t, x)$ is this a classical solution?

3. Show that each of the following is a symmetry of Burgers' equation:

   (a) $\overline{u} = u$, $\overline{t} = t + \tau$, $\overline{x} = x + \xi$,

   (b) $\overline{u} = -u$, $\overline{t} = -t$, $\overline{x} = x$,

   (c) $\overline{u} = 1/u$, $\overline{t} = x$, $\overline{x} = t$,

   (d) $\overline{u} = u + v$, $\overline{t} = t$, $\overline{x} = x + vt$.