MA 3426
Assignment 3
Due 19 March 2013

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1. Show that
$$u(x,y) = \frac{1}{4\pi}\log(x^2+y^2)$$
is a fundamental solution to the Laplace Equation in $\mathbf{R}^2$.
Hint: Write $\langle u_{xx}+u_{yy},\varphi\rangle$ as an integral. Split this integral into three pieces: one where $x^2+y^2<r_1^2$, one where $r_1^2 \le x^2+y^2 \le r_2^2$, and one where $r_2^2 < x^2+y^2$. Apply Green's Second Identity to the second of these, and let r_1 and r_2 tend to 0 and ∞, respectively.

2. Solve the initial value problem $u(t,x)=f(x)$ for the first order linear scalar equation
$$(t^2+1)u_t + (1+x^2)u_x = 0.$$
For which (t,x) is this a classical solution?

3. Show that each of the following is a symmetry of Burgers' equation:

   (a) $\overline{u}=u$, $\overline{t}=t+\tau$, $\overline{x}=x+\xi$,

   (b) $\overline{u}=-u$, $\overline{t}=-t$, $\overline{x}=x$,

   (c) $\overline{u}=1/u$, $\overline{t}=x$, $\overline{x}=t$,

   (d) $\overline{u}=u+v$, $\overline{t}=t$, $\overline{x}=x+vt$.