

Too Many Gold Medals?

John Stalker*

August 1, 2012

As I write these words¹, China has won 17 medals at the 2012 Summer Olympic games, of which 9, slightly over half, are gold medals.² Is it surprising that a country would win more gold medals than silver and bronze medals combined? That depends on the total number of medals won. In the 2008 games there six countries most of whose medals were gold. Panama, Tunisia and Cameroon each won a single gold medal and no others. Mexico won two gold medals and one bronze. Ethiopia won four golds, a silver and two bronze medals. China won 51 golds, 21 silvers and 28 bronze medals. No one should be particular surprised by the performances of Panama, Tunisia and Cameroon. If a country wins only a single medal then it is not particularly unlikely that it will be gold, and it follows logically that its gold medals will outnumber its other medals. It is not even surprising that there were three such countries, since there were 15 countries which one only a single medal. Ethiopia's performance might seem a little surprising, but we will soon see that it is not. China's performance so far in the 2012 games is more unusual, but not really remarkable. China's 2008 performance, however, is of an entirely different character. It is extremely unlikely that a country with 100 medals would, purely by chance, have the majority of them gold. There is almost certainly more to this story. It does not, however, follow that anything sinister is happening. There are multiple reasons why a country's medal totals might be unbalanced. I will discuss these later.

*School of Mathematics, Trinity College Dublin, Copyright 2012 John Stalker

¹31 August 2012

²Except for these 2012 results, all medal counts are taken from the Daily Telegraph's online lists. There are some judgment calls involved in compiling any such list, *e.g.* whether the 1992 "United Team" results are to be counted towards the Soviet Union or not. I have not sought to replace their judgment with mine on any of these questions.

Before proceeding further we need to be more precise about what we mean when we say that a medal total is unbalanced. What we would like is a single number, which we can calculate from the numbers of medals of each type won by a country, which tells us how unlikely it is that any unbalance has occurred by chance. Perhaps surprisingly, there is such a number. If the numbers of gold, silver and bronze medals are k , l and m then we set

$$n = k + l + m$$

and

$$\begin{aligned} s &= \frac{k^2 + l^2 + m^2 - kl - km - lm}{(k-l)^2 + (l-m)^2 + (m-k)^2} \\ &= \frac{3k^2 + 3l^2 + 3m^2 - n^2}{2n}. \end{aligned}$$

n is just the total number of medals. s is a measure of unbalance in the medal count. The three expressions for s are easily seen to be equivalent. From any of them we can see that s is zero if $k = l = m$. From the second of them we can see immediately that $s > 0$ unless $k = l = m$. So $s = 0$ means a completely balanced set of medal results. This is however not to be expected. Assuming each type of medal is equally likely and that medal results in different events are independent, the expected value of s is 1. Values of s below 1 represent unusually balanced medal results and values above 1 represent unusually unbalanced medal results. Note that the expected value of s does not depend on n , the number of medals. Still assuming each type of medal is equally likely and that medal results in different events are independent, the expected value of s is 1, we can, for any given number $\sigma \geq 0$, calculate the probability that $s \geq \sigma$. This probability depends on n in a complicated way, but it is approximately e^{-s} . The larger n is the better the approximation.

If we observe a large value of s then we must accept that one of two things has happened. Either our assumptions that each type of medal is equally likely and that medal results in different events are independent are correct but an event of very small probability has nonetheless caused an unusually large unbalance in the medal totals, or at least one of those two assumptions is false.

One thing to observe about s is that it scales linearly. In other words, if a country's number of gold, silver and bronze medals are all doubled then so is

s . One effect of this is that if some countries have a systematic bias in their medal distribution then this will become more apparent the more medals they win. We will see, in fact, that the largest values of s are generally to be found among the most successful countries.

Now that we have a simple quantitative way of discussing medal counts, we can return to the numbers above. For example, for Ethiopia in 2008, $k = 4$, $l = 1$, $m = 2$, $n = 7$, and $s = 1.0$. So s , our measure of unbalance, has exactly its expected value. For China in 2012, as of this writing, $k = 9$, $l = 5$, $l = 3$, $n = 17$, and $s = 28/17 \approx 1.65$. The probability of this degree of unbalance or larger occurring by chance is approximately $e^{-s} \approx 0.19$. This is an unusual result in the same sense that rolling a die and getting a six is an unusual result. By itself it is not worthy of note. China in 2008 is another matter entirely. There $k = 51$, $l = 21$, $m = 28$, $n = 100$, and $s = 739/100 = 7.39$. The probability of this occurring by chance is approximately $e^{-s} \approx 0.00062$. This isn't like rolling a die and getting a six. It's like rolling 4 dice and getting all sixes. Events like this should be rare, and indeed they are. I checked all participating countries in all Olympics going back to 1936 and found only three more unbalanced results. For Germany in 1952, $k = 0$, $l = 7$, $m = 17$, $n = 24$, and $s = 9.12$. For Australia in 1996, $k = 9$, $l = 9$, $m = 28$, $n = 46$, and $s = 7.85$. Both of these are unbalanced in the opposite way from China in 2008: Germany and Australia had disproportionately large numbers of bronze medals rather than gold medals. In fact, of the 24 medals Germany won in 1952, none were gold. The number s ignores such distinctions. It measures the extent of imbalance rather than its direction. There is one case of medal totals skewed towards gold in an even more extreme way than China's 2008 results: the United States in 1984³ with an almost unbelievable $s = 12.2241$.

Something interesting happens when we look at the medal distribution for individual countries over all Summer Olympic games rather than particular games. The medal statistics for the top ten countries are given in the table. $p = e^{-s}$ is approximately the probability of this level of unbalance occurring by chance in a world where medal outcomes are equally likely and independent.

³It should be noted that the 1976, 1980 and 1984 Summer Olympic Games were all subject to extensive boycotts. In the case of the 1984 games, the Soviet Union and most of its satellites boycotted the games.

| Country | k | l | m | n | s | p |
|---------------|-----|-----|-----|------|---------|---------|
| United States | 932 | 725 | 637 | 2294 | 29.9952 | 0.00000 |
| Soviet Union | 397 | 323 | 301 | 1021 | 7.4319 | 0.00059 |
| Great Britain | 211 | 258 | 253 | 722 | 2.7687 | 0.06274 |
| France | 192 | 210 | 235 | 637 | 2.1962 | 0.11122 |
| Italy | 191 | 157 | 175 | 523 | 1.6597 | 0.19020 |
| Germany | 189 | 218 | 239 | 646 | 2.9272 | 0.05354 |
| China | 163 | 122 | 106 | 391 | 6.6317 | 0.00132 |
| Hungary | 160 | 141 | 159 | 460 | 0.7457 | 0.47442 |
| East Germany | 153 | 130 | 128 | 411 | 1.4088 | 0.24445 |
| Sweden | 140 | 158 | 172 | 470 | 1.6426 | 0.19349 |

The most unbalanced medal totals are those for the United States, $s = 29.04$, and the Soviet Union, $s = 21.28$. the probabilities of these numbers occurring by chance are $p = 2.4 \times 10^{-13}$ and $p = 5.8 \times 10^{-10}$. This is like rolling 16 or 12 dice and getting all sixes. We can safely say this didn't happen by chance. China's totals are the third most unbalanced, but well behind the United States and the Soviet Union. For China, $k = 163$, $l = 117$, $m = 105$, $n = 385$, and $s = 7.3039$. The s value, and hence the probability, are close to those when we consider the 2008 games alone. Note that for all three countries the bias is towards gold medals.

It's clear that at least some of these variations in medal totals are not the result of random fluctuations. There are some systematic biases at work. It is worth taking the time to consider these in some detail.

- *Cheating*: Let's start with the most interesting one. We should not be surprised if some countries currently cheat or have done so in the past. It is hardly a secret, for example, that Soviet block Gymnastics judges colluded before the breakup of the Soviet Union. Gymnastics, due to the nature of the judging, is particularly susceptible to such manipulation. In the Soviet case the cheating was organised, but the same results could occur through the uncoordinated action of individuals. A gold medal is worth a large amount of money in sponsorship in country like the United States, enough to be worth bribing a judge or two if that can be done. If there are a large number of competitors from a particular country who are willing to do this, and if there are enough judges willing to be bribed, then this will shift the medal totals in a systematic way, even with any large scale organisation. I am not saying this has happened, just that this is one of the possibilities to consider.

There are other sports, however, where manipulation by officials, while not impossible, would be much more difficult than it is in Gymnastics. Most events in Athletics–Track and Field, for American readers–have much more clear cut results than Gymnastics competitions. This gives us some hope for identifying whether a country’s medal imbalance is partly due to dishonesty, by comparing imbalances in events where manipulation of results is difficult to those where it is relatively easy.

- *Underrepresentation of Strong Countries:* To compete in the Olympics one must first qualify, and the qualification procedures have a bias towards weaker countries. This is clearest in team sports like Baseball, where at most one team from each country participates, but it applies in varying degrees to most events. The effect is to eliminate from the games competitors from strong countries who are not quite good enough for a gold medal, but who might have achieved a silver or bronze. We should therefore expect the top medal winning countries to have values of $s > 1$, and this is what is observed. Of the top 10 all time medal winning countries, only Hungary has $s < 1$. While this effect is real, it is probably not large enough to explain much of the imbalance in medal totals. Baseball is not a typical Olympic event. Even very successful countries have space in most events for competitors who won’t even get a bronze medal. The most successful country of all time in terms of medals per competitor was East Germany. As we will see later, there is good evidence that this did affect East Germany’s medal distribution, but the country still had a relatively modest $s = 1.54$.
- *Investment Priorities:* Some countries may choose to concentrate their resources on a few people who seem sure to win a gold, at the expense of large numbers of others who might have a chance at a silver or bronze. Such a country would, if its predictions are correct, accumulate gold medals at the expense of silver and bronze medals. In theory the reverse is also possible, which would lead to a skew towards bronze medals. A country which is more interested in its total medal count rather than specifically gold medals could switch resources away from athletes once they seem sure of at least a bronze to try to bring others up to that level. The problem with either of these theories, but especially the second of them, is that they assume competitions are much more predictable than we generally perceive them to be. Athletes who seem sure of a

gold not infrequently go home without even a bronze and many gold medals are won by athletes who seemed unlikely to win any kind of medal. Predicting bronze medallists is even harder than predicting gold medallists. Because of the uncertainties of competition, we should expect this effect to exist and to contribute to medal imbalance, but we should probably not expect that contribution to be large.

- *Event Categories Dominated by a Few Countries:* Table Tennis is instructive. China and South Korea dominate the event. Only one other country has ever won a gold: Sweden with one gold. South Korea has three gold medals. China has 20. China and South Korea also account for most of the silver and bronze medals, but other countries have managed to pick up a few. The result is moderately large medal imbalances for the dominant countries. For China, $k = 20$, $l = 13$, $m = 8$, $n = 41$, and $s = 2.66$. Interestingly, the imbalance is larger for South Korea $k = 3$, $l = 2$, $m = 12$, $n = 17$ and $s = 5.35$. South Korea is good enough at Table Tennis to accumulate bronze medals, but has had limited success with silver and gold medals. A country which competes mainly in events in which it is dominant can expect a large medal imbalance. China's $s = 2.66$ in Table Tennis is not exceptionally large, but this is somewhat misleading. Imagine for a moment that China divided the Olympics into areas like Table Tennis, which it would dominate, and other areas in which it would not compete at all. To reach its historical total of 385 medals it would need 9 other areas. If it was as successful in each of these as in Table Tennis then it would have a combined $s = 26.6$, about halfway between the United States and the Soviet Union. This shows that even very large medal imbalances can be explained if two conditions are met: some events or categories of events are dominated by a small number of countries and a major country competes seriously mainly in those areas in which it is dominant.

It is not possible to establish definitively the existence or size of any of the above effects using only the historical medal counts, but we can look for clues. Gymnastics⁴, already mentioned, is a good place to start. The historical medal distributions for the top 10 countries are given in the following table.

⁴Only Artistic Gymnastics medals are counted, not Rhythmic or Trampoline. Including those would have little effect, due to the small number of medals awarded.

| Country | k | l | m | n | s | p |
|----------------|-----|-----|-----|-----|--------|---------|
| Soviet Union | 72 | 67 | 43 | 182 | 3.9615 | 0.01903 |
| United States | 29 | 34 | 31 | 94 | 0.2021 | 0.81699 |
| Japan | 28 | 31 | 33 | 92 | 0.2065 | 0.81341 |
| Romania | 24 | 19 | 25 | 68 | 0.4559 | 0.63389 |
| China | 22 | 15 | 14 | 51 | 1.1176 | 0.32705 |
| Switzerland | 16 | 19 | 13 | 48 | 0.5625 | 0.56978 |
| Hungary | 14 | 11 | 14 | 39 | 0.2308 | 0.79392 |
| Italy | 14 | 5 | 8 | 27 | 2.3333 | 0.09697 |
| Germany | 13 | 9 | 14 | 36 | 0.5833 | 0.55804 |
| Czechoslovakia | 12 | 13 | 10 | 35 | 0.2000 | 0.81873 |

The final column lists $p = e^{-s}$, which is approximately the probability of such a level of imbalance arising by chance in a world where all medals are equally likely and all events are independent. For example, Italy, with slightly more than half of its medals gold, has the second most unbalanced medal totals. We should expect to see this level of imbalance around 9.7% of the time, or roughly one time in ten. In a list of ten countries this is hardly surprising. The Soviet result is one we should expect only 1.9% of the time. We should be slightly surprised to see such a result occurring by chance, even in a table of ten countries. But the value $s = 3.9615$ is, as we will see, comparable to numbers in event categories where performance is much more objectively measurable. Gymnastics may be particularly susceptible to collusion among judges, but the evidence from the medal tables for such collusion is weak at best. Perhaps the rumours about Soviet block judges were really just rumours.

Diving is another event category where performance is somewhat subjective.

| Country | k | l | m | n | s | p |
|---------------|-----|-----|-----|-----|--------|---------|
| United States | 47 | 40 | 41 | 128 | 0.3359 | 0.71467 |
| China | 27 | 14 | 8 | 49 | 5.7755 | 0.00310 |
| Sweden | 6 | 8 | 7 | 21 | 0.1429 | 0.86688 |
| Germany | 5 | 9 | 9 | 23 | 0.6957 | 0.49875 |
| Soviet Union | 4 | 4 | 6 | 14 | 0.2857 | 0.75148 |
| Russia | 3 | 7 | 6 | 16 | 0.8125 | 0.44375 |
| Italy | 3 | 4 | 2 | 9 | 0.3333 | 0.71653 |
| Australia | 3 | 2 | 6 | 11 | 1.1818 | 0.30672 |
| East Germany | 2 | 2 | 3 | 7 | 0.1429 | 0.86688 |
| Mexico | 1 | 4 | 5 | 10 | 1.3000 | 0.27253 |

Here we see an anomalously high value of s for China, but not for any other countries. The probability that this has occurred by chance is very small, 0.3%, but we need to look at other event categories before suggesting that something is wrong with Diving judging.

Archery and Shooting are event categories in which the criteria for victory seem relatively straightforward. In Archery there is no evidence of any systematic unbalance.

| Country | k | l | m | n | s | p |
|---------------|-----|-----|-----|-----|--------|---------|
| South Korea | 16 | 9 | 5 | 30 | 3.1000 | 0.04505 |
| United States | 13 | 8 | 8 | 29 | 0.8621 | 0.42229 |
| Belgium | 10 | 5 | 3 | 18 | 2.1667 | 0.11456 |
| France | 6 | 11 | 7 | 24 | 0.8750 | 0.41686 |
| Great Britain | 2 | 2 | 5 | 9 | 1.0000 | 0.36788 |
| China | 1 | 5 | 1 | 7 | 2.2857 | 0.10170 |
| Soviet Union | 1 | 3 | 3 | 7 | 0.5714 | 0.56472 |
| Italy | 1 | 2 | 3 | 6 | 0.5000 | 0.60653 |
| Ukraine | 1 | 1 | 2 | 4 | 0.2500 | 0.77880 |
| Finland | 1 | 1 | 2 | 4 | 0.2500 | 0.77880 |

In Shooting there is one country, the United States, whose medal tallies are noticeably unbalanced.

| Country | k | l | m | n | s | p |
|---------------|-----|-----|-----|-----|--------|---------|
| United States | 50 | 29 | 24 | 103 | 5.5437 | 0.00391 |
| China | 19 | 11 | 12 | 42 | 1.3571 | 0.25740 |
| Soviet Union | 17 | 16 | 18 | 51 | 0.0588 | 0.94287 |
| Sweden | 15 | 22 | 18 | 55 | 0.6727 | 0.51031 |
| Norway | 13 | 8 | 11 | 32 | 0.5938 | 0.55225 |
| Great Britain | 12 | 15 | 16 | 43 | 0.3023 | 0.73910 |
| Italy | 10 | 9 | 11 | 30 | 0.1000 | 0.90484 |
| France | 8 | 11 | 7 | 26 | 0.5000 | 0.60653 |
| Germany | 8 | 8 | 6 | 22 | 0.1818 | 0.83375 |
| Russia | 7 | 11 | 8 | 26 | 0.5000 | 0.60653 |

The level of the United States' unbalance in Shooting is similar to China's in Diving, and is in the same direction, a bias towards gold medals.

The results for Athletics are particularly interesting, in part due to the large number of medals awarded in this event category.

| Country | k | l | m | n | s | p |
|---------------|-----|-----|-----|-----|---------|---------|
| United States | 311 | 238 | 188 | 737 | 15.5753 | 0.00000 |
| Soviet Union | 64 | 55 | 74 | 193 | 1.4041 | 0.24558 |
| Great Britain | 49 | 80 | 62 | 191 | 3.8063 | 0.02223 |
| Finland | 48 | 35 | 30 | 113 | 2.2920 | 0.10106 |
| East Germany | 38 | 36 | 35 | 109 | 0.0642 | 0.93780 |
| Kenya | 22 | 27 | 19 | 68 | 0.7206 | 0.48647 |
| Poland | 22 | 17 | 13 | 52 | 1.1731 | 0.30941 |
| Germany | 19 | 38 | 42 | 99 | 4.5758 | 0.01030 |
| Australia | 19 | 24 | 25 | 68 | 0.4559 | 0.63389 |
| Sweden | 19 | 21 | 40 | 80 | 5.0375 | 0.00649 |

Sweden's results are systematically biased towards bronze medals. Germany's results are biased against gold medals. Britain, peculiarly, seems to be biased towards silver. It is tempting to dismiss that as a statistical fluke, but Britain's combined medal totals in other event categories are also biased towards silver, so it seems likely that this is a real, if rather odd, phenomenon. What is particularly striking, however, is the massive imbalance in US medals. It is nearly impossible that this could have occurred purely by chance. There are probably a number of effects at work here, but the most important is probably the domination of this event category by the United States. The US has more gold medals than all other countries in the top

ten combined. This also explains why several other countries see their medal distribution shifted towards the bronze. US domination of Athletics is not as complete as Chinese domination of Table Tennis, but the total number medals awarded is much larger. Since s scales linearly with n a smaller bias towards gold produces a larger value of s in this event category.

In fact the number of medals awarded in Athletics, and the level of US dominance, is such that it accounts for about half of the unbalance in US medal totals in the Summer Olympics as a whole. More precisely, if we remove Athletics medals from consideration then the United States' value of s drops from 29.0392 to 14.8673. Most of the remaining imbalance can be explained by Swimming.

| Country | k | l | m | n | s | p |
|---------------|-----|-----|-----|-----|---------|---------|
| United States | 214 | 155 | 120 | 489 | 13.8466 | 0.00000 |
| Australia | 56 | 54 | 58 | 168 | 0.0714 | 0.93106 |
| East Germany | 38 | 32 | 22 | 92 | 2.1304 | 0.11879 |
| Hungary | 23 | 23 | 17 | 63 | 0.5714 | 0.56472 |
| Japan | 20 | 21 | 21 | 62 | 0.0161 | 0.98400 |
| Netherlands | 18 | 17 | 18 | 53 | 0.0189 | 0.98131 |
| Great Britain | 15 | 22 | 28 | 65 | 1.9538 | 0.14173 |
| Germany | 14 | 23 | 35 | 72 | 4.6250 | 0.00980 |
| Soviet Union | 12 | 21 | 26 | 59 | 2.5593 | 0.07736 |
| Sweden | 8 | 14 | 13 | 35 | 0.8857 | 0.41242 |

After removing both Athletics and Swimming, the United States' medal statistics are $k = 405$, $l = 335$, $m = 331$, $n = 1071$, $s = 4.8515$ and $p = 0.0078$.

Where does the Soviet Union's Medal imbalance come from? Part of it can be explained by its dominance of Weightlifting.

| Country | k | l | m | n | s | p |
|---------------|-----|-----|-----|-----|---------|---------|
| Soviet Union | 39 | 21 | 2 | 62 | 16.5645 | 0.00000 |
| China | 24 | 11 | 8 | 43 | 5.0465 | 0.00643 |
| United States | 16 | 16 | 11 | 43 | 0.5814 | 0.55912 |
| Bulgaria | 12 | 16 | 8 | 36 | 1.3333 | 0.26360 |
| France | 9 | 3 | 3 | 15 | 2.4000 | 0.09072 |
| Turkey | 8 | 1 | 1 | 10 | 4.9000 | 0.00745 |
| Germany | 6 | 7 | 7 | 20 | 0.0500 | 0.95123 |
| Greece | 6 | 5 | 4 | 15 | 0.2000 | 0.81873 |
| Italy | 5 | 4 | 5 | 14 | 0.0714 | 0.93106 |
| Unified Team | 5 | 4 | 0 | 9 | 2.3333 | 0.09697 |

In fact, if we remove both Weightlifting and Gymnastics then the Soviet Union's medal counts become balanced, with $s = 0.7297$.

One of the most striking biases towards gold medals in any event category is to be found in Rowing.

| Country | k | l | m | n | s | p |
|---------------|-----|-----|-----|-----|---------|---------|
| East Germany | 33 | 7 | 8 | 48 | 13.5625 | 0.00000 |
| United States | 31 | 31 | 22 | 84 | 0.9643 | 0.38126 |
| Great Britain | 24 | 17 | 9 | 50 | 3.3800 | 0.03405 |
| Germany | 23 | 16 | 13 | 52 | 1.5192 | 0.21888 |
| Romania | 19 | 10 | 8 | 37 | 2.7838 | 0.06180 |
| Soviet Union | 12 | 20 | 11 | 43 | 1.6977 | 0.18311 |
| Italy | 10 | 13 | 12 | 35 | 0.2000 | 0.81873 |
| Australia | 10 | 10 | 12 | 32 | 0.1250 | 0.88250 |
| Canada | 9 | 14 | 12 | 35 | 0.5429 | 0.58109 |
| France | 7 | 14 | 13 | 34 | 1.2647 | 0.28232 |

More than three quarters of East Germany's 48 medals were gold. There are currently only 14 Rowing events per Olympics and East Germany participated in only 5 Summer Olympic Games. The total number of races it participated in is not exactly 70, because events are occasionally added or removed from the competition. I have not been able to determine the exact number, but it is clear that East Germany won around half the races it competed in and received medals in the vast majority. It is likely that it would have won even more medals, mostly silver and bronze, if it had been able to field more teams, but there is a limit of one boat from each country in each race. East German Rowing is perhaps the best historical example of a skew towards gold caused by underrepresentation of strong countries.

It is instructive to look again at the top gold medal winning countries of the five most recent Summer Olympic Games. For 1992:

| Country | <i>k</i> | <i>l</i> | <i>m</i> | <i>n</i> | <i>s</i> | <i>p</i> |
|---------------|----------|----------|----------|----------|----------|----------|
| Unified Team | 45 | 38 | 29 | 112 | 1.7232 | 0.17849 |
| United States | 37 | 34 | 37 | 108 | 0.0833 | 0.92004 |
| Germany | 33 | 21 | 28 | 82 | 1.3293 | 0.26467 |
| China | 16 | 22 | 16 | 54 | 0.6667 | 0.51342 |
| Spain | 14 | 7 | 2 | 23 | 4.7391 | 0.00875 |
| Cuba | 14 | 6 | 11 | 31 | 1.5806 | 0.20584 |
| South Korea | 12 | 5 | 12 | 29 | 1.6897 | 0.18458 |
| Hungary | 11 | 12 | 7 | 30 | 0.7000 | 0.49659 |
| France | 8 | 5 | 16 | 29 | 3.3448 | 0.03527 |
| Australia | 7 | 9 | 11 | 27 | 0.4444 | 0.64118 |

For 1996:

| Country | <i>k</i> | <i>l</i> | <i>m</i> | <i>n</i> | <i>s</i> | <i>p</i> |
|---------------|----------|----------|----------|----------|----------|----------|
| United States | 49 | 32 | 25 | 106 | 4.3113 | 0.01342 |
| Russia | 26 | 21 | 16 | 63 | 1.1905 | 0.30408 |
| Germany | 20 | 18 | 27 | 65 | 1.0308 | 0.35673 |
| China | 16 | 27 | 12 | 55 | 3.2909 | 0.03722 |
| France | 15 | 7 | 15 | 37 | 1.7297 | 0.17733 |
| Italy | 13 | 10 | 12 | 35 | 0.2000 | 0.81873 |
| Australia | 9 | 9 | 28 | 46 | 7.8478 | 0.00039 |
| Cuba | 9 | 8 | 8 | 25 | 0.0400 | 0.96079 |
| Ukraine | 9 | 2 | 12 | 23 | 3.4348 | 0.03223 |
| South Korea | 7 | 15 | 5 | 27 | 3.1111 | 0.04455 |

For 2000:

| Country | <i>k</i> | <i>l</i> | <i>m</i> | <i>n</i> | <i>s</i> | <i>p</i> |
|---------------|----------|----------|----------|----------|----------|----------|
| United States | 37 | 24 | 33 | 94 | 1.4149 | 0.24295 |
| Russia | 32 | 28 | 29 | 89 | 0.1461 | 0.86410 |
| China | 28 | 16 | 14 | 58 | 2.9655 | 0.05153 |
| Australia | 16 | 25 | 17 | 58 | 1.2586 | 0.28405 |
| Germany | 13 | 17 | 26 | 56 | 2.3750 | 0.09301 |
| France | 13 | 14 | 11 | 38 | 0.1842 | 0.83176 |
| Italy | 13 | 8 | 13 | 34 | 0.7353 | 0.47936 |
| Netherlands | 12 | 9 | 4 | 25 | 1.9600 | 0.14086 |
| Cuba | 11 | 11 | 7 | 29 | 0.5517 | 0.57596 |
| Great Britain | 11 | 10 | 7 | 28 | 0.4643 | 0.62858 |

For 2004:

| Country | k | l | m | n | s | p |
|---------------|-----|-----|-----|-----|--------|---------|
| United States | 36 | 39 | 27 | 102 | 1.1471 | 0.31757 |
| China | 32 | 17 | 14 | 63 | 4.4286 | 0.01193 |
| Russia | 28 | 26 | 38 | 92 | 1.3478 | 0.25980 |
| Australia | 17 | 16 | 16 | 49 | 0.0204 | 0.97980 |
| Japan | 16 | 9 | 12 | 37 | 1.0000 | 0.36788 |
| Germany | 13 | 16 | 21 | 50 | 0.9800 | 0.37531 |
| France | 11 | 9 | 13 | 33 | 0.3636 | 0.69514 |
| Italy | 10 | 11 | 11 | 32 | 0.0312 | 0.96923 |
| South Korea | 9 | 12 | 9 | 30 | 0.3000 | 0.74082 |
| Great Britain | 9 | 9 | 13 | 31 | 0.5161 | 0.59683 |

For 2008:

| Country | k | l | m | n | s | p |
|---------------|-----|-----|-----|-----|--------|---------|
| China | 51 | 21 | 28 | 100 | 7.3900 | 0.00062 |
| United States | 36 | 37 | 34 | 107 | 0.0654 | 0.93667 |
| Russia | 23 | 21 | 29 | 73 | 0.7123 | 0.49050 |
| Great Britain | 19 | 13 | 15 | 47 | 0.5957 | 0.55115 |
| Germany | 16 | 10 | 15 | 41 | 0.7561 | 0.46950 |
| Australia | 14 | 15 | 17 | 46 | 0.1522 | 0.85884 |
| South Korea | 13 | 10 | 8 | 31 | 0.6129 | 0.54178 |
| Japan | 9 | 6 | 10 | 25 | 0.5200 | 0.59452 |
| Italy | 8 | 9 | 10 | 27 | 0.1111 | 0.89484 |
| Ukraine | 7 | 5 | 15 | 27 | 3.1111 | 0.04455 |

There are a few things to note here. Except for Greece in 2004, the hosting country always made the top ten list. In each of those cases the hosting country had a medal distribution which was more unbalanced than expected, *i.e.* $s > 1$. In addition, the unbalance was always in the same direction, towards gold. This unbalance was mild in the case of Australia in 2000 and extreme in the case of China in 2008. Spain in 1992 and the United States in 1996 had less unbalanced distributions, but still too unbalanced to be explained plausibly by chance. Going further back, the pattern becomes less clear. There are examples of clear unbalance, like the 1984 Los Angeles games, but there are also some examples of balanced medal totals from the host country, such as the 1960 Rome Olympics. So the bias towards gold for the host country in recent games could be statistical noise, or it could

represent a real but recent phenomenon. This question seems to merit further investigation.

Equally interesting is the growing imbalance in China's medal totals. In 1992 China had $s = 0.667$, lower than the expected value. This level of unbalance is, as its p value suggests, likely to occur more often than not purely by chance. In each of 1996, 2000, and 2004 the level of unbalance was considerably greater. Such levels of unbalance would be expected to occur randomly 4%, 5% or 1% of the time, respectively. The level of unbalance in 2008 was much more extreme, with only a 0.06% chance of occurring by chance. This is, as explained earlier, not unprecedented, but it is extremely rare. Part of the growth in s values is explained by increasing numbers of total medals won, since s scales like n , but this can not explain all of the change, since even the ration s/n is growing. Interestingly, China's medals were skewed towards silver in 1996 and towards gold thereafter. This would seem to be consistent with an explanation based on prioritised investment in athletes expected to win a gold medal.

This has been a rather long essay, longer than I had originally intended, so perhaps it is worth summarising its principal conclusions.

- The distribution of countries' medals into gold, silver and bronze at a particular games or in a particular event category are not random. Medal counts can be skewed towards gold or bronze, and arguably silver, to an extent nearly impossible to attribute to random variation.
- There are multiple explanations for such biases, most of which are not sinister.
- These biases are clearest among the strongest countries, although that is partly a consequence of having more data to analyse.
- Perhaps surprisingly, Gymnastics and Diving do not show unusually large biases in individual countries' medal distributions compared to event categories where the criteria for victory appear to be more objective.
- Overall medal unbalance is often the consequence of unbalance in a small number of event categories, *e.g.* Athletics and Swimming for the United States or Gymnastics and Weightlifting for the Soviet Union.

- China's skew towards gold is a real phenomenon, is recent, and is not unprecedented.
- There may be host country effects, which would help to explain the United States' skew in 1984 and China's in 2008, but the evidence for these is inconclusive.
- The qualifications procedures in many events are biased against stronger countries, which results in a skew towards gold. The evidence suggests that this is, however, a relatively small effect.