MA 3421
Assignment 5
Due 21 November 2012

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1. It was shown in lecture that if $\sum_{k,l} |\alpha_{k,l}|^2 < \infty$ then the linear transformation $A\colon \ell^2 \to \ell^2$ defined by $(A\xi)_k = \sum_l \alpha_{k,l}\xi_l$ is compact. The condition $\sum_{k,l} |\alpha_{k,l}|^2 < \infty$ is sufficient for compactness, but not necessary. To show this, consider the operator A with

$$\alpha_{k,l} = \begin{cases} k^{-s} & \text{if } k = l, \\ 0 & \text{if } k \neq l, \end{cases}$$

where $s \geq 0$.

   (a) For which values of s is A bounded?

   (b) For which values of s is A compact?

   (c) For which values of s is A symmetric?

   (d) For which values of s is $\sum_{k,l} |\alpha_{k,l}|^2 < \infty$?

2. Let L and R be the left and right unilateral shift operators on ℓ^2,

$$(L\xi)_n = \xi_{n+1} \qquad (R\xi)_n = \begin{cases} 0 & \text{if } n = 1, \\ \xi_{n-1} & \text{if } n > 1. \end{cases}$$

   (a) Is L compact?

(b) Is R compact?

(c) Is L symmetric?

(d) Is R symmetric?

(e) What are the eigenvalues and eigenvectors of L?

(f) What are the eigenvalues and eigenvectors of R?