MA 3421
Assignment 2
Due 17 October 2012

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1. This problem and all the others on this problem set relate to a pair of linear transformations, $L\colon \ell^p \to \ell^p$ and $R\colon \ell^p \to \ell^p$, called the left and right unilateral shift, respectively.[1]

$$(L\xi)_n = \xi_{n+1} \qquad (R\xi)_n = \begin{cases} 0 & \text{if } n = 1, \\ \xi_{n-1} & \text{if } n > 1. \end{cases}$$

   Prove that L and R are bounded, and hence continuous, for all $1 \leq p \leq \infty$. Compute their norms.

2. What is

   (a) L^m?

   (b) R^m?

   (c) LR?

   (d) RL?

3. Show that

[1] Conventions vary about whether the sequences in ℓ^p begin with the zeroeth term or the first. For purposes of this problem, consider them as beginning with the first.

(a) L is surjective, but not injective.

(b) R is injective, but not surjective.

4. Prove that for any $\xi \in \ell^p$

$$\lim_{m\to\infty} L^m \xi = 0$$

if $1 \leq p < \infty$.