MA 3421
Assignment 1
Due 10 October 2012

`Id: 3421-f2012-1.m4,v 1.1 2012/10/03 20:48:03 john Exp john`

1. Show that $B(T)$, the set of bounded functions on an arbitrary set T, with the usual norm
$$\|x\| = \sup_{t \in T} |x(t)|,$$
is a Banach space, and therefore that ℓ^∞ is a Banach space.

2. Show that (c), the space of convergent sequences, with the usual norm
$$\|\xi\| = \sup_n |\xi_n|,$$
is a Banach space. The simplest way to do this is to use the result of the previous problem and the subspace criterion proved in lecture.

3. Show that in any normed space addition and scalar multiplication are continuous. Show also that the norm function and metric functions are continuous.