MA 2325
Assignment 6
Due 16 December 2009

`Id: 2325-0910-6.m4,v 1.2 2009/12/28 13:18:03 john Exp john`

1. Evaluate
$$\int_{-\pi}^{\pi} \frac{A\cos\theta + B\sin\theta + C}{a\cos\theta + b\sin\theta + c}\, d\theta$$
by contour integration, where $a^2 + b^2 < c^2$.

2. Evaluate
$$\sum_{-\infty}^{\infty} \frac{1}{1+n^2}$$
by contour integration.
Hint: Consider the integral of the function
$$f(z) = \pi\cot(\pi z)(1+z^2)^{-1}.$$
on a square centred at 0 with side length an odd integer.

3. Show that the function
$$u(x,y) = \sum_{j=0}^{n/2} (-1)^j \frac{n!}{(2j)!(n-2j)!} x^{n-2j} y^{2j}$$
is harmonic. By a theorem proved in class, there is a holomorphic function f such that $u(x,y)$ is the real part of $f(x+iy)$. Find f.