

MA 2325  
Assignment 5  
Due 2 December 2009

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1. Let

$$f(z) = \frac{\exp(2\pi i \xi z)}{1+z^2}, \quad g(z) = \frac{1}{1+z^3},$$
$$h(z) = \frac{\tan(z)}{z^2}, \quad k(z) = \frac{\sin(z)}{z}$$

Find all poles of  $f$ ,  $g$ ,  $h$  and  $k$ .

2. With  $f$ ,  $g$ ,  $h$  and  $k$  as above, find the orders of all their poles.
3. With  $f$ ,  $g$ ,  $h$  and  $k$  as above, find the residues at each pole.
4. Draw the path

$$\gamma(t) = \frac{2 \cos t}{1 + \sin^2 t} (1 + i \sin t) \quad 0 \leq t \leq 2\pi.$$

What are the winding numbers about the points  $1$ ,  $-1$ ,  $i$ ,  $-i$ ? You needn't lift any paths or compute any integrals, just use the picture you've drawn.