MA 2325
Assignment 4
Due 18 November 2009

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1. Sine, cosine and tangent are defined by

$$\sin(z) = \sum_{k=0}^{\infty} (-1)^k \frac{z^{2k+1}}{(2k+1)!} \qquad \cos(z) = \sum_{k=0}^{\infty} (-1)^k \frac{z^{2k}}{(2k)!}$$

   and

$$\tan(z) = \frac{\sin(z)}{\cos(z)}.$$

   Find the power series for $\tan(z)$ up through the z^7 term.

2. Find the power series for the function

$$f(z) = (1-z)^{-m}.$$

   Hint: Differentiation gives

$$f'(z) = m(1-z)^{-m-1} = m(1-z)^{-1} f(z)$$

   or

$$zf'(z) + mf(z) = f'(z).$$

   Use the formula for differentiation of power series to determine the coefficients of the power series for f.

3. Suppose that f is continuous in an open subset of the complex plane containing the real interval $[a,b]$ and that $\gamma\colon [a,b] \to \mathbf{C}$ is defined by $\gamma(t) = t$. Prove that

$$\int_\gamma f(z)dz = \int_a^b f(x)dx.$$

The integral on the left is a contour integral and the integral on the right is an ordinary real integral.
Hint: This isn't hard, just important.

4. Verify by computing the integral that Cauchy's Theorem holds for the function $f(z) = z^2$ and the triangle

$$T = \{x + iy \in \mathbf{C}\colon x \geq 0, y \geq 0, x + y \leq 1\},$$

i.e. show that

$$\int_{\partial T} f(z)\, dz = 0.$$