MA 2325
Assignment 3
Due 4 November 2009

Id: 2325-0910-3.m4,v 1.3 2009/11/01 12:13:13 john Exp john

1. It was shown in lecture that continuous functions on closed intervals are always uniformly continuous. On open intervals, including infinite intervals, this may not be true.

   (a) Show that $f(x) = x^2$ is *not* uniformly continuous on $\mathbf{R}$.

   (b) Show that $g(x) = \sin(x)$ *is* uniformly continuous on $\mathbf{R}$.

2. Compute, from the definition, the winding number of the path

$$\gamma_{n,w}(t) = \exp(2\pi i n t) + w \quad 0 \leq t \leq 1$$

   about w, where n is an integer.

3. Compute the contour integral

$$\int_{\gamma_{n,w}} \frac{dz}{z - w}$$

   from the definition, where the path $\gamma_{n,w}$ is the path defined in the preceding problem.

4. Show that for any closed path γ and point w not on γ,

$$\int_{\gamma} \frac{dz}{z - w} = 2\pi i n(\gamma, w).$$