MA 2325
Assignment 3
Due 4 November 2009

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1. It was shown in lecture that continuous functions on closed intervals are always uniformly continuous. On open intervals, including infinite intervals, this may not be true.

   (a) Show that $f(x) = x^2$ is *not* uniformly continuous on $\mathbf{R}$.

   (b) Show that $g(x) = \sin(x)$ *is* uniformly continuous on $\mathbf{R}$.

2. Compute, from the definition, the winding number of the path

$$\gamma_{n,w}(t) = \exp(2\pi int) + w \quad 0 \le t \le 1$$

   about w, where n is an integer.

3. Compute the contour integral

$$\int_{\gamma_{n,w}} \frac{dz}{z-w}$$

   from the definition, where the path $\gamma_{n,w}$ is the path defined in the preceding problem.

4. Show that for any closed path γ and point w not on γ,

$$\int_{\gamma} \frac{dz}{z-w} = 2\pi i n(\gamma, w).$$