MA 2325
Assignment 2
Due 21 October 2009

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1. In lecture it was proved that if both $\sum_{j=0}^{\infty} a_j$ and $\sum_{k=0}^{\infty} b_k$ are absolutely convergent then $\sum_{l=0}^{\infty} c_l$ is convergent where $c_l = \sum_{j=0}^{l} a_j b_{l-j}$, and that

$$\sum_{l=0}^{\infty} c_l = \left(\sum_{j=0}^{\infty} a_j\right)\left(\sum_{k=0}^{\infty} b_k\right).$$

   Prove that $\sum_{l=0}^{\infty} c_l$ converges absolutely.
   Hint: The most straightforward way to prove this is to use the theorem that every bounded increasing sequence of real numbers is convergent.

2. There is a power series $\sum_{k=0}^{\infty} b_k z^k$ such that

$$(\exp(z) - 1)\sum_{n=0}^{\infty} b_k z^k = z.$$

   Find b_k for $k = 0, 1, \ldots, 7$.

3. Prove that for real y,

$$\exp(iy) = \cos(y) + i\sin(y)$$

   and that for real x and y,

$$\exp(x + iy) = \exp(x)\cos(y) + i\exp(x)\sin(y).$$

4. Prove the algebraic identity used in lecture

$$\begin{aligned}(s-w)^n-(z-w)^n &= n(s-z)(z-w)^{n-1}\\ &\quad+(s-z)^2\sum_{k=0}^{n-2}(n-k-1)(s-w)^k(z-w)^{n-k-2}.\end{aligned}$$

Hint: Start with the special case $w=0$. Use induction, possibly more than once.