

XMA44481

FACULTY OF SCIENCE

SCHOOL OF MATHEMATICS

JS Maths
SS Maths
SS TP

Trinity Term 2011

COURSE 4448

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ATTEMPT THREE QUESTIONS
All questions have equal weight (20 points)

1. (20 points)

(a) (5 points) Consider a spacetime whose metric, in coordinates t, r, θ, φ has

$$g_{tt} = -c^2 e^{a(r)} \quad g_{rr} = e^{b(r)} \quad g_{\theta\theta} = r^2 \quad g_{\varphi\varphi} = r^2 \sin^2 \theta$$

for some functions a and b and constant c . All other components are zero. Suppose I asked you to determine for which a and b the metric is flat. How would you proceed? Don't actually calculate anything; just indicate what you would do.

(b) (5 points) The same question, except that I ask when the metric is vacuum.

(c) (5 points) Compute the non-zero Christoffel symbols.

(d) (5 points) If you were asked to compute the Riemann curvature tensor, which components would you need to compute from the definition, and which could you get by symmetry considerations?

2. (20 points)

(a) (5 points) For which r and s do the partial derivatives of an (r, s) tensor form an $(r, s + 1)$ tensor?

(b) (5 points) List the defining properties of the covariant derivative.

(c) (5 points) Find the difference of the mixed covariant derivatives

$$T_{;cd}^{ab} - T_{;dc}^{ab}$$

for a $(2, 0)$ tensor T , in terms of T and the Riemann curvature tensor.

(d) (5 points) State the Bianchi identities.

3. (20 points) The Kasner metric has the form

$$g_{tt} = -c^2 \quad g_{xx} = t^{2\alpha} \quad g_{yy} = t^{2\beta} \quad g_{zz} = t^{2\gamma}$$

where α, β and γ are constants such that $\alpha + \beta + \gamma = 1$ and $\alpha^2 + \beta^2 + \gamma^2 = 1$

(a) (5 points) Show that there is no redshift between stationary observers.

- (b) (5 points) Write down the Hamiltonian form of the equations of motion of a chargeless point mass.
- (c) (5 points) What are the energy and momenta? Which of these are conserved?
- (d) (5 points) Find three linearly independent Killing vector fields for this spacetime.

4. (20 points)

- (a) (4 points) Write down the Maxwell Equations for a 4-potential Φ_a , electromagnetic field F_{ab} and charge-current density J^a in curved space.
- (b) (4 points) What is the energy-momentum-stress tensor of the field?
- (c) (6 points) Calculate this tensor for the Reissner-Nordström solution

$$g_{tt} = -c^2 \Delta \quad g_{rr} = \Delta^{-1} \quad g_{\theta\theta} = r^2 \quad g_{\varphi\varphi} = r^2 \sin^2 \theta$$

$$\text{where } \Delta = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \text{ and } F_{tr} = -F_{rt} = Q/r^2$$

- (d) (3 points) What are the Einstein Field Equations?
- (e) (3 points) Does the Reissner-Nordström solution satisfy the Einstein Field Equations?