MA 4448
Assignment 2
Due 23 February 2011

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1. With spherical coordinates (t, r, θ, φ), let $g_{(0)} = \partial/\partial t$, $g_{(1)} = \partial/\partial r$, $g_{(2)} = r^{-1}\partial/\partial\theta$, and $g_{(3)} = (r\sin\theta)^{-1}\partial/\partial\varphi$.

   (a) Show that $\{g_{(0)}, g_{(1)}, g_{(2)}, g_{(3)}\}$ is a local frame.

   (b) Compute the metric coefficients in this frame, and show that this frame is a tetrad.

   (c) Compute the commutation coefficients, defined by

   $$[g_{(b)}, g_{(c)}] = \gamma^a_{bc} g_{(a)}.$$

2. Suppose $X = X^a\partial/\partial x^a$ and $Y = Y^b\partial/\partial x^b$ are vector fields and $\alpha = \alpha_c dx^c$ is a 1-form. Prove that

   $$(d\alpha)(X, Y) = X\alpha(Y) - Y\alpha(X) - \alpha([X, Y]).$$

3. Suppose $\{g_{(0)}, g_{(1)}, g_{(2)}, g_{(3)}\}$ is a general frame, *i.e.* that it's values at x are a basis for $T_x\mathcal{M}$. Let $\{g^{(0)}, g^{(1)}, g^{(2)}, g^{(3)}\}$ be the dual frame, *i.e.* its values at x are the dual basis for $T^*_x\mathcal{M} = \Omega^1_x\mathcal{M}$. Then $\{g^{(0)} \wedge g^{(1)} = -g^{(1)} \wedge g^{(0)}, g^{(0)} \wedge g^{(2)} = -g^{(2)} \wedge g^{(0)}, g^{(0)} \wedge g^{(3)} = -g^{(3)} \wedge g^{(0)}, g^{(1)} \wedge g^{(2)} = -g^{(2)} \wedge g^{(1)}, g^{(1)} \wedge g^{(3)} = -g^{(3)} \wedge g^{(1)}, g^{(2)} \wedge g^{(3)} = -g^{(3)} \wedge g^{(2)}\}$ is, at each point x, a basis for $\Omega^2_x\mathcal{M}$. What are $\{dg^{(0)}, dg^{(1)}, dg^{(2)}, dg^{(3)}\}$ in this basis?
   Note: You may use the result of the previous problem even if you didn't succeed in proving it.