MA 4448
Assignment 1
Due 2 February 2011

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1. Figure out how the ordinary derivatives $A^{i_1\cdots i_r}_{j_1\cdots j_s,k}$ of an arbitrary (r,s) tensor $A^{i_1\cdots i_r}_{j_1\cdots j_s}$ transform under coordinate changes.
   If you get lost in the indices then can do this for a $(2,2)$, although the general case is actually easier.

2. Compute the Christoffel symbols of Minkowski space in

   (a) Cartesian coordinates: $(x^0, x^1, x^2, x^3) = (t, x, y, z)$. For your sanity and mine, write indices as t, x, y and z rather than 0, 1, 2 and 3. So the metric is $g_{tt} = -c^2$, $g_{xx} = g_{yy} = g_{zz} = 1$ and all other components are zero.

   (b) Spherical Coordinates: $(x^0, x^1, x^2, x^3)(t, r, \theta, \varphi)$. The metric in these coordinates is $g_{tt} = -c^2$, $g_{rr} = 1$, $g_{\theta\theta} = r^2$, $g_{\varphi\varphi} = r^2 \sin^2\theta$ and all other components are zero.

3. Read about raising and lowering of indices in Section 3.1.1 of George Ellis' notes. It's fairly simple. Then,

   (a) Show that $g^{ij}_{;k}$ is zero.

   (b) What do raising and lowering correspond to in the coordinate-free description of tensors?

(c) Prove that we get the same result from differentiating and then raising/lowering as we get from raising/lowering and then differentiating. Differentiating here means covariant differentiation.