

MAU34804

Lecture 24

2026-03-30

# This week

As with most sophister maths modules this one doesn't meet in the last week of term, and doesn't present new material in the second to last week, i.e. this week.

The plan for this week is:

Monday (today): structured revision

Tuesday (tomorrow): question and answer session

Wednesday: more q&a if we run out of time on Tuesday, otherwise no class.

# Today

## Module summary/review

- new topics
- main theorems
- main applications

## Exam information

- structure
- relation to mock exam
- relation to past exams
- revision suggestions

## (Probably) unfamiliar mathematical concepts

In addition to standard material from compulsory modules there were various things you probably hadn't seen before elsewhere:

- Correspondences and hemicontinuity
- Convexity, concavity, quasiconvexity and quasiconcavity
- Simplices and simplicial complexes
- The role of positivity in linear algebra

# Main theorems

The main theorems are the three fixed point theorems, Brouwer, Kakutani, and Banach, and the Berge maximum theorem.

Memorising the proofs is probably not a good use of time, but you certainly need to know their statements and you should be able to use them.

Other theorems or propositions mostly belong to one or more of these three categories:

- Needed for the proof of one of the main theorems, e.g. Sperner's lemma
- Application of one of the main theorems, e.g. the von Neumann minimax theorem
- Needed for handling examples, e.g. hemicontinuity criteria

There are more of these auxiliary results than the main theorems, and they are easier to examine.

Knowing what isn't true is also important!

# Main applications

The main areas we applied the main theorems to are:

- production models (Leontief)
- exchange models (Walras)
- game theory (von Neumann, Nash)

This is a maths module so most of the exam is mathematics rather than economics.

# Exam structure

There are four multipart questions, of which you have to do three. Points for parts are indicated on the paper.

I've tried to make the parts as independent as I could, but with limited success.

Unless otherwise indicated you can use any result from the notes or lectures anywhere.

Try to be explicit about what you're using, e.g. " $Y$  is closed and bounded, and so is compact, and  $\Phi(\mathbf{x})$  is closed for all  $\mathbf{x} \in X$  so by a pair of theorems in the notes  $\Phi$  is upper hemicontinuous if and only if its graph is closed."

In other words, you don't need to cite theorems by name or number and you don't need to state them precisely unless asked but I should be able to figure out which ones you're using and see whether you're aware of and have checked their hypotheses.

I can give partial credit to incorrect answers, but only if I understand what you're trying to do.

# Revision resources

- Mock exam (posted on Friday)
- Past exams (on exams office website)
- Notes (on module website)
- Slides (on module website)

# Mock exam

On the module web page there is a mock exam. At some point there will also be solutions.

The mock exam has the same structure as the real exam.

It has a similar mix of definitions, examples, proofs, etc.

It does not, except accidentally, cover the same topics as the real exam.

We've covered more topics than I can examine so some topics need to be selected at random. The mock and real exams are different random selections from the same set of topics.

The fact that a topic appears on the mock exam doesn't mean it is more or less likely to appear on the real exam than another topic which doesn't appear.

There is no point in memorising solutions to questions on the mock exam. The reasons I'm posting them are:

- so you can check whether your answers are correct, and
- so you can see roughly what level of detail I'm looking for in your answers.

## Past exams

All of the past exams are from David Wilkins, except for the 2020 exam which was from me, but was a 24 take home exam.

David's old papers are still fairly useful though. The topics are almost the same, and the notation and terminology as well.

There are some important differences though. David sometimes gave a couple of pages of definitions and statements of theorems at the end of the paper but I'm not doing that this year.

David was much more likely than I am to ask for proofs of things proved in the notes or in class.

I'm more likely to ask for proofs of unseen, but hopefully easier, theorems.

I'm also more likely to ask questions comparing two theorems.

The proportion of definitions and statements looks fairly similar, but I'm more likely to ask for counter-examples.

# Notes

From the notes you should know the main definitions, theorems, and examples. Exact phrasing isn't important, but it is important to have all the hypotheses there. And conclusions, of course, but it's less common to omit those. Proofs are less important, particularly long ones. If you're looking at proofs, concentrate on the main ideas rather than tricks. Chapter 1 is meant to be review of material from other modules. No question is intended to examine it but you may need it to answer questions intended to test later topics. There are a few sections I skipped, one in Chapter 2, three in Chapter 4, and two in Chapter 7. Those are not examinable. Other than those you are responsible for the contents of the notes though, even parts not covered in lectures.

# Slides

Everything covered in lecture is in theory examinable, including material not covered in the notes, but some things are less examinable in practice.

In particular, the material on positive matrices I covered between Chapters 5 and 6 is examinable.

Elsewhere I mostly followed the notes, but some things were done differently in lecture. In particular I mostly tried to do different examples. Knowing lots of examples is good, so use the fact that you have more of them.

# Tomorrow

Tomorrow I'll take questions, so think about what you want to ask.

I can't answer questions about what's on the exam, but I can answer questions about which material is examinable.

Asking about how to solve questions on the mock exam is allowed, but largely pointless, since I'll post solutions.