MA 342H
Assignment 3
Due 29 March 2018

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1. Compute the Euler-Lagrage equations for

   (a) the first order Lagrangian in one independent variable

   $$L(x, u, u_x) = x\sqrt{1 + u_x^2},$$

   (b) the second order Lagrangian in one independent variable

   $$L(x, u, u_x, u_{xx}) = \frac{u_{xx}}{1 + u_x^2},$$

   and

   (c) the first order Lagrangian in three independent variables

   $$L(t, x, y, u, u_t, u_x, u_y) = -\frac{1}{2}u_t^2 + \frac{1}{2}u_x^2 + \frac{1}{2}u_y^2 + \frac{1}{2}u^2.$$

2. Find the vector fields associated to the one parameter groups of transformations on $\mathbf{R}^3 \times \mathbf{R}$, with coordinates t, x, y, u,

   (a)

   $$\tilde{t} = t\cosh s + x\sinh s, \quad \tilde{x} = t\sinh s + x\cosh s, \quad \tilde{y} = y, \quad \tilde{u} = u$$

   and

   (b)

   $$\tilde{t} = e^{2s}t, \quad \tilde{x} = e^{2s}x, \quad \tilde{y} = e^{2s}y, \quad \tilde{u} = e^{-s}u.$$

3. Find the first order prolongations of the vector fields from the previous problem.

4. Show that the vector fields above are symmetries of the first order Lagrangian

$$L(t, x, y, u, u_t, u_x, u_y) = -\frac{1}{2}u_t^2 + \frac{1}{2}u_x^2 + \frac{1}{2}u_y^2$$

and that the first, but not the second, of them is also a symmetry of

$$L(t, x, y, u, u_t, u_x, u_y) = -\frac{1}{2}u_t^2 + \frac{1}{2}u_x^2 + \frac{1}{2}u_y^2 + \frac{1}{2}u^2.$$

5. Find the conservation laws associated to

   (a) The transformation

   $$\tilde{t} = t\cosh s + x\sinh s, \quad \tilde{x} = t\sinh s + x\cosh s, \quad \tilde{y} = y, \quad \tilde{u} = u$$

   for the Lagrangian

   $$L(t, x, y, u, u_t, u_x, u_y) = -\frac{1}{2}u_t^2 + \frac{1}{2}u_x^2 + \frac{1}{2}u_y^2 + \frac{1}{2}u^2,$$

   and

   (b) the transformation

   $$\tilde{t} = e^{2s}t, \quad \tilde{x} = e^{2s}x, \quad \tilde{y} = e^{2s}y, \quad \tilde{u} = e^{-s}u.$$

   for the Lagrangian

   $$L(t, x, y, u, u_t, u_x, u_y) = -\frac{1}{2}u_t^2 + \frac{1}{2}u_x^2 + \frac{1}{2}u_y^2.$$