MA 342H
Assignment 2
Due 14 March 2018

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1. Show that
$$k(x) = -\frac{\exp(-\|x\|)}{4\pi\|x\|}$$
is a fundamental solution for
$$p(\partial) = \partial_1^2 + \partial_2^2 + \partial_3^2 - 1$$
in $\mathbf{R}^3$.
Hint: Use Green's second identity on the region
$$\int_\Omega (u \operatorname{div} \operatorname{grad} v - v \operatorname{div} \operatorname{grad} u) = \int_{\partial\Omega} (u \operatorname{grad} v - v \operatorname{grad} u) \cdot n$$
where $\partial\Omega$ is the boundary of Ω and n is the exterior unit normal there. You'll want
$$\Omega = \{x \in \mathbf{R}^3 \colon \epsilon < \|x\| < \rho\}.$$
$u = k$ and $v = \varphi \in \mathcal{D}(\mathbf{R}^3)$.

2. (a) Find a fundamental solution for the differential operator
$$p(\partial) = \partial^2 + 2\partial + 2$$
on $\mathbf{R}$.

   (b) Use the fundamental solution you just found to solve the initial value problem for the inhomogeneous equation
$$u''(x) + 2u'(x) + 2u(x) = f(x), \quad u(0) = \alpha, \quad u'(0) = \beta.$$
If you didn't manage to find a fundamental solution then just take as given that there is one and call it k.