MA 342H
Assignment 1
Due 14 February 2018

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1. For the linear first order scalar partial differential equation

$$\begin{aligned}(-x-y-z)\frac{\partial u}{\partial w}(w,x,y,z)+(w-y+z)\frac{\partial u}{\partial x}(w,x,y,z)\\ +(w+x-z)\frac{\partial u}{\partial y}(w,x,y,z)+(w-x+y)\frac{\partial u}{\partial z}(w,x,y,z)=0,\end{aligned}$$

with initial conditions

$$u(1,x,y,z)=f(x,y,z)$$

   (a) Find the non-characteristic points on the initial hypersurface.

   (b) Write down and solve the characteristic equations.

   (c) Solve the differential equation. Note that you only need your solution to make sense near the non-characteristic part of the initial hypersurface.

2. (a) Solve Burgers' Equation

$$\frac{\partial u}{\partial t}(t,x)+u(t,x)\frac{\partial u}{\partial x}(t,x)=0$$

with initial data

$$u(0,x)=-\frac{x}{\sqrt{1+x^2}}.$$

You should get a quartic equation for u with coefficients which are polynomials in t and x. You needn't solve this quartic.

(b) For the solution you obtained in the previous part, find $u(t, 0)$. For which values of t does uniqueness fail?

3. Solve the initial value problem

$$u(0, x) = f(x)$$

for the first order scalar equation

$$\frac{\partial u}{\partial t}(t, x) + \frac{1}{2}\left(\frac{\partial u}{\partial x}(t, x)\right)^2 + \frac{1}{2}x^2 = 0.$$

Note that fully eliminating parameters is not possible with f unspecified. The best you can do it two equations with one extra variable.