MA 3425
Assignment 3
Due 30 October 2012

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1. Show that for solutions of the homogeneous Heat Equation, $u_t - k u_{xx} = 0$, on an interval $[a, b]$ with Neumann boundary conditions at the endpoints, $u_x(t, a) = u_x(t, b) = 0$, that the integral
$$\int_a^b u(t, x)\, dx$$
is constant.
Hint: Differentiate under the integral sign and integrate by parts.

2. Show, by means of an example, that the integral from the preceding problem need not be constant if Dirichlet boundary conditions, $u(t, a) = u(t, b) = 0$, are imposed instead of Neumann conditions. *Hint:* You have seen a counterexample already.

3. Suppose that u satisfies the initial value problem $u(0, x) = f(x)$ for the homogeneous Heat Equation on the real line. Suppose further that f, in addition to being bounded and continuous, is such that $|f(x)|^2$ is integrable. Show that
$$\sup_{-\infty < x < \infty} |u(t, x)|,$$
which we know by the maximum principle is a bounded function of t, tends to zero as t tends to infinity.
Hint: You need an inequality for integrals. Several of these were discussed in lecture, and in a paper linked from the course web site.