MA 3425
Assignment 2
Due 16 October 2012

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1. For which p, q, r is the scaling $\overline{u} = \lambda^p u$, $\overline{t} = \lambda^q t$, $\overline{x} = \lambda^r x$ a symmetry of the cubic non-linear wave equation

$$u_{tt} - c^2 u_{xx} + u^3 = 0?$$

2. Show that for solutions of the cubic non-linear wave equation the energy

$$\int_{-\infty}^{\infty} \epsilon(t, x)\, dx$$

with energy density

$$\frac{1}{2}u_t^2 + \frac{c^2}{2}u_x^2 + \frac{1}{4}u^4$$

is constant.
Hint: Of the proofs of energy conservation for the ordinary wave equation presented in class, only the last one, the one with the trapezoids, can be adapted to this problem.