MA 3425
Assignment 1
Due 2 October 2012

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1. For each of the following, say whether it is a scalar equation or system, give the order, and state whether it is linear or non-linear.

   (a) The 1+3 Dimensional Klein-Gordon Equation

   $$u_{tt} - u_{xx} - u_{yy} - u_{zz} + u = 0$$

   (b) The Incompressible Euler Equations

   $$\begin{aligned} u_t + uu_x + vu_y + wu_z + p_x &= 0 \\ v_t + uv_x + vv_y + wv_z + p_y &= 0 \\ w_t + uw_x + vw_y + ww_z + p_z &= 0 \\ u_x + v_y + w_z &= 0 \end{aligned}$$

   (c) The Korteweg-de Vries Equation

   $$u_t - u_{xxx} + 6uu_x = 0$$

2. $u(t,x) = \cos(ckt)\cos(kx)$ is called a standing wave.

   (a) Verify that it satisfies the homogeneous Wave Equation

   $$u_{tt} - c^2 u_{xx} = 0$$

   for any k.

   (b) Write it in the form $u(t,x) = \varphi(x+ct) + \psi(x-ct)$.

3. Solve the initial value problem $u(0,x) = f(x)$, $u_t(0,x) = g(x)$ for the Wave Equation,

   (a) with $f(x) = (1+x^2)^{-1}$ and $g(x) = 0$,

   (b) with $f(x) = 0$ and $g(x) = (1+x^2)^{-1}$.