MAU34215 Assignment 1
Due 1 October 2025

1. What are the orders of the following differential equations? Which of them are linear?

   - Helmholtz equation:
   $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + u = 0,$$
   - Biharmonic equation:
   $$\frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4} = 0,$$
   - Eikonal equation:
   $$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 - 1 = 0,$$
   - Euler-Tricomi equation:
   $$\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial y^2} = 0,$$
   - Minimal surface equation:
   $$\left[1 + \left(\frac{\partial u}{\partial y}\right)^2\right] \frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + \left[1 + \left(\frac{\partial u}{\partial x}\right)^2\right] \frac{\partial^2 u}{\partial y^2} = 0,$$
   - Aller-Lytkov equation:
   $$a\frac{\partial^3 w}{\partial t \partial x^2} + d\frac{\partial^2 w}{\partial x^2} - \frac{\partial w}{\partial t} = 0.$$

2. (a) Show that if the initial data f and g are infinitely differentiable then so is the solution u to the initial value problem for the wave equation.

   (b) Show the following sort of converse: If u is infinitely differentiable for $t > s$ then f and g are infinitely differentiable.

3. Suppose that a solution of the wave equation on the interval $[a, b]$ satisfies the Robin boundary conditions

$$\frac{\partial u}{\partial x}(t, a) - \alpha u(t, a) = 0, \quad \frac{\partial u}{\partial x}(t, b) + \beta u(t, b) = 0.$$

   (a) Show that

$$\frac{\alpha c^2}{2} u(t, a)^2 + \int_a^b \left[\frac{1}{2} \left(\frac{\partial u}{\partial t}(t, x) \right)^2 + \frac{c^2}{2} \left(\frac{\partial u}{\partial x}(t, x) \right)^2 \right] dx + \frac{\beta c^2}{2} u(t, b)^2$$

   is independent of t.

   (b) Prove uniqueness of solutions to the initial value problem with Robin boundary conditions under the assumption that α and β are non-negative.

   (c) Does your argument for the preceding part work without that assumption?