

MAU34215 Assignment 3
Due 2026-11-04

1. (a) Suppose u is a solution to the initial value problem

$$\frac{\partial u}{\partial t}(t, x) - k \frac{\partial^2 u}{\partial x^2}(t, x) = 0, \quad u(0, x) = f(x)$$

where f is bounded and continuous. Prove that

$$\int_{-\infty}^{+\infty} |u(t, x)| dx \leq \int_{-\infty}^{+\infty} |f(x)| dx.$$

- (b) With hypotheses as above prove that the inequality above is strict if and only if there are points x_1 and x_2 where $f(x_1) > 0$ and $f(x_2) < 0$.

2. (a) Find a polynomial equation satisfied by solutions of the initial value problem

$$u(0, x) = -\frac{x}{\sqrt{1+x^2}}$$

for Burgers' equation.

- (b) Is there a classical solution to the initial value problem from the previous part which is valid for all values of t and x ?

Hint: You may find it helpful to consider the values of $\partial u / \partial x$ on the line $x = 0$.