MAU34215 Assignment 1
Due 2026-09-30

1. For the following equations list the dependent and independent variables, give their order, whether they are linear or non-linear, and whether they are ordinary or partial differential equations.

   In some cases the answer may be different for special values of the parameters. In those cases you can just give the answer for the generic case.

   (a) Helmholtz:
   $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + k^2 u = 0$$

   (b) Eikonal:
   $$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 1$$

   (c) Picard-Fuchs:
   $$\frac{d^2 f}{dj^2} + \frac{1 - 1968j + 2654208j^2}{4j^2(1-1728j)^2} f = 0$$

   (d) Painlevé II:
   $$\frac{d^2 y}{dt^2} = 2y^3 + ty + \alpha$$

   (e) Lane-Emden:
   $$\xi \frac{d^2\theta}{d\xi^2} + 2\frac{d\theta}{d\xi} + \xi\theta^n = 0$$

   (f) Tzitzeica:
   $$w\frac{\partial^2 w}{\partial x \partial y} - \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} - w^3 + 1 = 0$$

   (g) Camassa-Holm:
   $$\frac{\partial u}{\partial t} + 2\kappa\frac{\partial u}{\partial x} - \frac{\partial^3 u}{\partial t \partial x^2} + 3u\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial x}\partial^2 u \partial x^2 + u\frac{\partial^3 u}{\partial x^3} = 0$$

2. Which of the symmetries we found for the wave equation are also symmetries of the Klein-Gordon equation?

3. The following relate to the Lorentz transformations L_v from the notes and solutions in the region $ct > |x|$ in the plane.

   (a) Show that a function u is invariant under the Lorentz transformations if and only if it is of the form

   $$u(t, x) = \varphi(c^2t^2 - x^2).$$

   for some function φ on the positive real numbers.

   (b) Show that a continuously differentiable function u is invariant under the Lorentz transformations if and only if

   $$x\frac{\partial u}{\partial t} + c^2t\frac{\partial u}{\partial x} = 0$$

   everywhere.

   (c) Find all Lorentz invariant solutions.