

MA 3421 Assignment 4, Due 18 October 2018

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1. Suppose $P \in \mathcal{L}(E)$ is a projection on a normed space E , i.e. that $P^2 = P$. Suppose also that $P \neq 0$. Show that $\|P\| \geq 1$.
Hint: Proof by contradiction is your friend.

2. Suppose that

$$\lim_{n \rightarrow \infty} x_n = y$$

in a Banach algebra E with identity element e . If there is no n for which x_n is invertible then show that y is not invertible.

3. Suppose that (f_1, f_2, \dots) is a sequence of bounded functions on \mathbf{R} converging uniformly to g . Suppose also that

$$\inf_{r \in \mathbf{R}} |f_n(t)| = 0$$

for each n . Show that

$$\inf_{r \in \mathbf{R}} |g(t)| = 0.$$

4. Show that in any inner product space

$$\|x + y\|^2 + \|x - y\|^2 = 2\|x\|^2 + 2\|y\|^2$$

for all vectors x, y .

5. Give an example of a normed space E and vectors $x, y \in E$ such that the equation above does not hold.

6. Suppose that the *real* vector space E is an inner product space. Show that

$$(x|y) = \left\| \frac{x+y}{2} \right\|^2 - \left\| \frac{x-y}{2} \right\|^2.$$