MA 3421 Assignment 3, Due 4 October 2018

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1. Recall that norms $r, s\colon E \to \mathbf{R}$ are called equivalent if there are $\mu, \nu \in \mathbf{R}^+$ such that, for all $x \in E$,

   $$\mu r(x) \le s(x) \le \nu r(x).$$

   Show that equivalence of norms is, as the name suggests, an equivalence relation.

2. Show that the $l^p(n)$ and $l^q(n)$ norms on $\mathbf{K}^n$ are equivalent for any $1 \le p, q < \infty$. Try to obtain the sharpest values for μ and ν possible.

3. Suppose F is a subspace of a Banach space E. Show that F, with the norm inherited from E, is a normed space and that it is a Banach space if and only if it is closed.

4. Suppose F is a subspace of a Banach space E. Define $p\colon E/F \to \mathbf{R}$ by

   $$p(X) = \inf_{x \in X} \|x\|.$$

   Show that p is a seminorm on E/F and that is a norm if and only if F is closed.

5. Let P be the subspace of $C([a, b])$ consisting of polynomials restricted to $[a, b]$. As usual, assume $a < b$. Show that P is a proper subspace, but not a closed subspace.
   Note: You may use, without proof, the Weierstrass approximation theorem.