MA 3421 Assignment 2, Due 27 September 2018

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1. Suppose E is a vector space and $p: E \to \mathbf{R}$ is a semi-norm. Let

   $$F = \{x \in E: p(x) = 0\}.$$

   Show that

   (a) F is a subspace.

   (b) there is a unique function $q: E/F \to \mathbf{R}$ such that $x \in X$ implies

   $$q(X) = p(x).$$

   (c) q is a norm.

2. Compute $\|A\|$ where $A \in \mathcal{L}(l^p(2))$, $p \geq 1$, and

   $$A((\xi_1, \xi_2)) = (\xi_2, 0).$$

   Compute $\|A^2\|$ and $\|A\|^2$.

3. Show that the following analogues for normed spaces of familiar properties of the absolute value in $\mathbf{K}$, some of which have been already been used without comment in the notes and in lecture.

   (a) The norm is a continuous function.

   (b) $x_n \to y$ if and only if $\|x_n - y\| \to 0$.

   (c) Every Cauchy sequence, and hence every convergent sequence, is bounded.

   The proofs are in all cases the same as the proofs in $\mathbf{R}$, with absolute values replaced by norms where required.

4. Show that if

   $$\lim_{j \to \infty} A_j = C$$

   in $\mathcal{L}(F, G)$ and

   $$\lim_{j \to \infty} B_j = D$$

in $\mathcal{L}(E, F)$ then

$$\lim_{j \to \infty} A_j B_j = CD$$

in $\mathcal{L}(E, G)$.