

MA 3421 Assignment 1, Due 20 September 2018

Revision: 1.0

Date: 2018-09-13 11:30:15+01

Check <http://www.maths.tcd.ie/~stalker/3421/> for most recent version.

1. Suppose that $f: E \rightarrow F$ is a function from one metric space to another. Show that the following statements are equivalent:

- (a) If $x_n \rightarrow y$ in E then $f(x_n) \rightarrow f(y)$ in F .
- (b) For all $\epsilon > 0$ and $y \in E$ there is a $\delta > 0$ such that, for all $x \in E$,

$$\|x - y\| < \delta$$

implies

$$\|f(x) - f(y)\| < \epsilon.$$

2. Show that convergence in (s) and pointwise convergence are equivalent.
3. Show that convergence in $C([a, b])$ and pointwise convergence are *not* equivalent for $a < b$.
4. Suppose that E is equipped with the discrete metric. Show that all subsets are both open and closed and that a subset is compact if and only if it is finite.
5. Are the sequences e_1, e_2, \dots a basis for the sequence space (s) ? Recall that e_j is the sequence whose j 'th element is 1 and all other elements are 0.
6. Show that the quotient of (c) by its subspace (c_0) is one-dimensional.