MAU23205 2021-2022 Practice Problem Set 6

1. Solve the initial value problem $y(x_0) = y_0$ for the separable equation $y'(x) = -y(x)^{1/3}$. Try to get a solution valid on as large an interval as possible.

2. For the system

$$\frac{dx}{dt} = -x + ay + x^2y \qquad \frac{dy}{dt} = b - ay - x^2y$$

   (a) Find the equilibria, if any.
       Note: your answer will depend on the values of a and b.

   (b) In the cases were there is a unique equilibrium, find its linearisation at that equilibrium.
       Warning: It is not correct just to drop the terms of degree greater than one from the polynomials! Doing so would give the linearisation at $(0,0)$, but $(0,0)$ is not generally where the equilibrium is located.

   (c) For which values of a and b is the linearisation stable? For which values is it strictly stable?

3. The system

$$x'(t) = -y(t) \qquad y'(t) = x(t) - y(t) + x(t)^2y(t)$$

   has an equilibrium at $(0,0)$. For which values of a is

$$V(x, y) = x^2 + axy + by^2$$

   a strict Lyapunov function for this equilibrium.