MAU23205 2021-2022 Practice Problem Set 5

1. Let
$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$
For each of the following A find a symmetric C such that $A^T B + BA + C = O$. For which A's is the B you found positive definite?

   (a)
$$A = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$$

   (b)
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

   (c)
$$A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$$

2. Find the Green's function for the second order scalar equation
$$x''(t) + 2x'(t) + 2x(t) = 0.$$

3. Find the fundamental matrix W for
$$A(t) = \begin{bmatrix} 1/t & t \\ 0 & 1 \end{bmatrix}$$

4. The Bessel equation of order ν is
$$x^2 y''(x) + xy'(x) + (x^2 - \nu^2)y(x) = 0.$$
Show that there is no non-zero power series solution unless ν is an integer, in which case there is one whose first non-zero term is the $x^{|\nu|}$ term. Where does it converge?